Electroproduction of medium- and heavy-mass hypernuclei

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Outline:

Introduction
Formalism of DWIA
– impulse approximation in optimal factorisation,
– elementary amplitude in a general two-component form
– optimum on-shell approximation
Results: effects in the cross sections from
– kinematics (Fermi motion, the mass),
– kaon distortion,
– ΛN interaction.
Predictions for the JLab experiments on ${}^{40}_{\Lambda}$Ca, ${}^{48}_{\Lambda}$Ca, and ${}^{208}_{\Lambda}$Pb.
Summary

More information in Phys. Rev. C **106**, 044609 (2022) and **108**, 024615 (2023).

Introduction – Why and how to study hypernuclei?

- a direct investigation of the YN interaction is quite complicated, *e.g.* Λp scattering in the liquid-hydrogen target

$$\gamma + p \to K^+ + \Lambda$$
$$\Lambda + p \to p + \pi^- + p'$$

  [see J. Rowley etal (CLAS Collaboration), Phys. Rev. Lett. 127, 272303 (2021)], information on the bare YN interaction is limited (Nijmegen model)

- the γ-ray and reaction spectroscopy of Λ hypernuclei can provide information on the spin-dependent part of an effective ΛN interaction

$$\mathrm{V}_{\Lambda\mathrm{N}} = \mathrm{V}_0 + \mathrm{V}_\sigma\, \vec{s}_\Lambda \cdot \vec{s}_\mathrm{N} + \mathrm{V}_\Lambda\, \vec{\ell}_{\Lambda\mathrm{N}} \cdot \vec{s}_\Lambda + \mathrm{V}_\mathrm{N}\, \vec{\ell}_{\Lambda\mathrm{N}} \cdot \vec{s}_\mathrm{N} + \mathrm{V}_\mathrm{T}\, \mathrm{S}_{12}$$

- in hypernuclei one can study Λ–Σ mixing: $\Lambda + \mathrm{N} \longleftrightarrow \Sigma^0 + \mathrm{N}$; charge symmetry breaking in mirror hypernuclei: ${}^4_\Lambda\mathrm{He} - {}^4_\Lambda\mathrm{H}$, ${}^{16}_{\Lambda}\mathrm{N} - {}^{16}_{\Lambda}\mathrm{O}$; and non mesonic weak decays of Λ: $\Lambda\mathrm{N} \longrightarrow \mathrm{nN}$

Introduction – the γ-ray (classical) spectroscopy

- an example of γ-ray transitions in the hadron-induced reaction

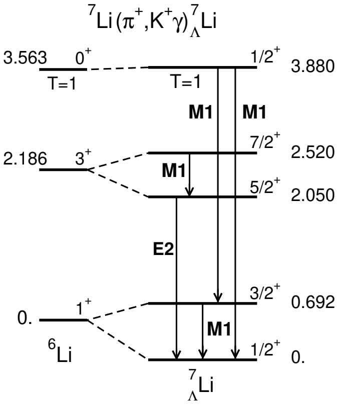
  $\pi^+ + {}^7\text{Li} \rightarrow \text{K}^+ + {}^7\text{Li}^*_\Lambda \rightarrow {}^7\text{Li}_\Lambda + \gamma + \text{K}^+$

  with K^+ and γ detected in coincidence
- γ's are measured by Ge detector array with a high resolution $\approx$ 1 keV
- analysis of spectra was done for p-shell hypernuclei $\rightarrow$ the effective interaction $V_{\Lambda N}$ was obtained

  [D. J. Millener, Nucl.Phys. A 804, 84 (2008)]
- limitations:
  – only the low-lying states are determined (below the particle emision threshold)

  – the ground-state binding energy cannot be determined

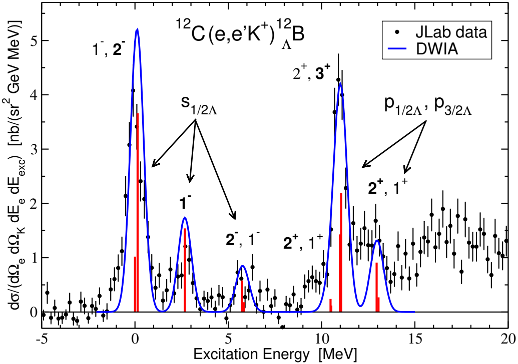
Λ hypernucleus is a relatively long living system $\approx 10^{-10}s$

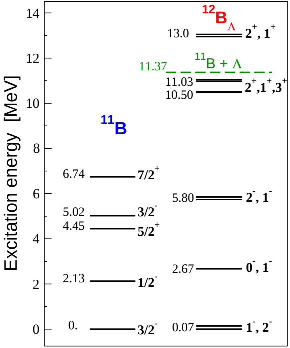


by H. Tamura

Introduction – the reaction spectroscopy

- the reaction spectroscopy allows to study also the high-excited states *e.g.* here those with Λ in the p orbit
- missing mass spectrum in $e + {}^{12}\mathrm{C} \to e' + K^+ + {}^{12}\mathrm{B}^*_\Lambda$, where e' and K^+ are detected
- the energy resolution is worse than in the γ spectroscopy but below 1 MeV

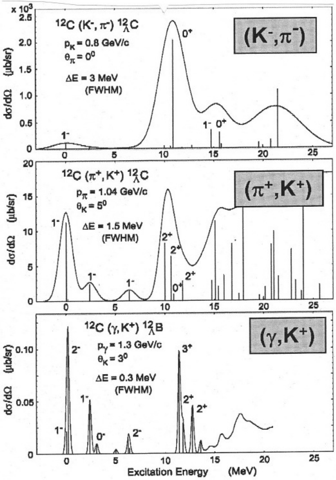




M. Iodice at al, Phys. Rev. Lett. **99**, 052501 (2007)

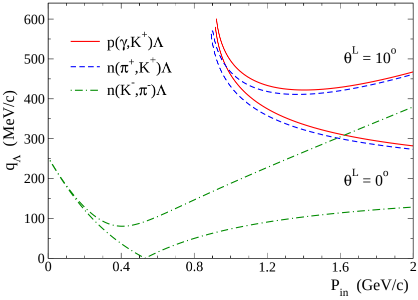
Introduction – electroproduction of hypernuclei

- one can achieve a better energy resolution than in the π^+ and K^- induced reactions



by Hashimoto and Tamura

- large momentum transfer: $q_\Lambda > 250$ MeV/c



- one can also study the **reaction mechanism**: kinematical effects, factorization, off-shell effects, kaon distortion,...; and test the elementary amplitudes

Introduction – electroproduction of hypernuclei

- other characteristics of electroproduction:
  - the electro-magnetic interaction is well known and weak ($\alpha \approx 1/137$)
    $\rightarrow$ the one-photon exchange approximation is very good which simplifies description to production by virtual photons: $\mathrm{A}(\gamma_{\mathrm{v}}, \mathrm{K}^{+})\mathrm{H}$
  - production is on the proton
    $\rightarrow$ other hypernuclei than in (π^{+}, K^{+}) and (K^{-}, π^{-}) reactions
  - if K^{0} is detected then production goes on the neutron in (γ, K^{0})
  - a strong spin-flip $\rightarrow$ the highest-spin states in multiplets dominate

- typical kinematics in experiments:
  - a small electron scattering angle and small photon virtuality $Q^2 = -q^2$
    $\rightarrow$ this is to achieve sufficiently big virtual photon flux (Γ),
  - kaon is detected along the photon direction (z-axis) at a very small kaon angle (θ_K)

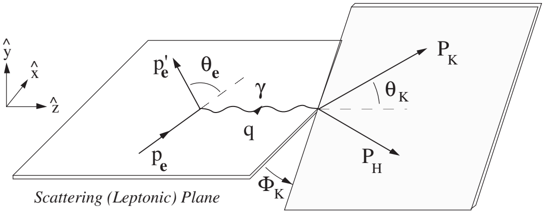
- To obtain reliable information from hypernucleus electroproduction data we need to understand the reaction mechanism well,
  e.g. to estimate systematic uncertainties due to various approximations.

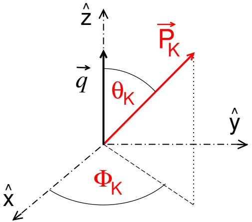
Formalism – kinematics in electroproduction

- production on the proton:

  $$\mathrm{e}(\mathrm{p_e}) + {}^{A}\mathrm{Z}(\mathrm{P_A}) \longrightarrow \mathrm{e}'(\mathrm{p}'_\mathrm{e}) + \mathrm{K}^+(\mathrm{P_K}) + {}^{A}_{\Lambda}(\mathrm{Z}-1)(\mathrm{P_H})$$

  photon energy and momentum: $E_\gamma = E_e - E'_e$ and $\vec{q} = \vec{p}_e - \vec{p}'_e$
- in the laboratory frame ($\vec{\mathrm{P}}_\mathrm{A} = 0$):





- kinematics in an experiment: $\mathrm{E_e}$, E'_e, θ_e, θ_{Ke}, Φ_K

  $\rightarrow \epsilon$, Γ, $\mathrm{q} = \mathrm{p_e} - \mathrm{p}'_\mathrm{e}$, $Q^2 = -q^2$, $|\vec{\mathrm{P}}_\mathrm{K}|$, θ_K ...

Formalism – the cross section

- the unpolarized triple-differential cross section

$$\frac{d^3\sigma}{dE'_e\, d\Omega'_e\, d\Omega_K} = \Gamma \left[\frac{d\sigma_T}{d\Omega_K} + \epsilon_L \frac{d\sigma_L}{d\Omega_K} + \epsilon \frac{d\sigma_{TT}}{d\Omega_K} + \sqrt{2\epsilon_L(1+\epsilon)} \frac{d\sigma_{TL}}{d\Omega_K} \right]$$

  - Γ and ϵ are virtual photon flux factor and polarization, respectively, $\epsilon_L = \epsilon\ Q^2/E_\gamma^2$, and $d\sigma/d\Omega_K$ are separated cross sections
  - $d\sigma_T$ dominates due to very small $Q^2 \leftrightarrow$ photoproduction ($Q^2 = 0$)
  - $d\sigma_{TL}$ is important even for small Q^2:

the cross sections in $^{12}C(e, e'K^+)^{12}_{\Lambda}B$ with $Q^2 = 0.06\ (GeV/c)^2$

E	J_H^P	$\theta_{K\gamma}^{lab}$	$d\sigma$	$d\sigma_T$	$\epsilon_L d\sigma_L$	$\epsilon d\sigma_{TT}$	$\sqrt{2\epsilon_L(1+\epsilon)}d\sigma_{TL}$	$d^3\sigma$
0.0	1^-	1.8	36.768	42.505	0.251	0.040	-6.028	0.640
0.116	2^-	1.8	127.898	148.198	1.083	0.364	-21.747	2.227

$\rightarrow$ the transverse-longitudinal interference terms contribute more than 10%

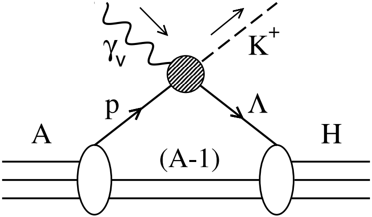
$\rightarrow$ there is a difference between photo- and electroproduction calculations

Formalism – the impulse approximation

- production by virtual photons: $\gamma_\mathrm{v}(\mathrm{q}) + \mathrm{A}(\mathrm{P_A}) \longrightarrow \mathrm{H}(\mathrm{P_H}) + \mathrm{K}^+(\mathrm{P_K})$
- in IA the production goes on individual protons which is well justified for large momenta: $|\vec{\mathrm{q}}| \geq 1$ GeV/c $\Rightarrow \lambda_B \leq 0.2$ fm

- the many-particle matrix element

$$\langle \Psi_\mathrm{H} | \sum_{i=1}^{Z} \mathcal{X}_\gamma \mathcal{X}_\mathrm{K}^* \mathcal{J}_\mu^i(\mathrm{p}_\Lambda, \mathrm{P}_K, \mathrm{p}_p^i, \mathrm{q}) \, | \Psi_\mathrm{A} \rangle$$



Ψ_H, Ψ_A – hypernucleus and nucleus nonrelativistic wave functions

$\mathcal{J}_\mu^i$ – elementary production amplitude in two-component form

$\mathcal{X}_\gamma$ – photon plane wave

$\mathcal{X}_\mathrm{K}$ – kaon distorted wave in eikonal approximation:

a good approximation as $|\vec{\mathrm{P}}_K| \approx 1$ GeV/c and the KN interaction is weak

Formalism – the optimal factorization

- the elementary amplitude is integrated over the proton momentum $\vec{p}_p$ in the many-particle matrix element
- assuming a constant effective proton momentum the elementary amplitude can be taken out of the integral and we get the hypernucleus-production amplitude in the optimal-factorization approximation (OFA)

$$\begin{aligned} \mathcal{T}_\mu &= Z\,\mathrm{Tr}\left[\mathcal{J}_\mu(\vec{P}_K, \vec{p}_{eff}, \vec{q}\,) \int d^3\xi\, e^{\mathrm{i}B\vec{\Delta}\cdot\vec{\xi}}\, \mathcal{X}^*_{\mathrm{K}}(\vec{p}_{\mathrm{K}}, B\vec{\xi}\,) \right. \\ &\quad \left. \times \int d^3\xi_1 \ldots d^3\,\xi_{A-2}\; \Phi_{\mathrm{A}}(\vec{\xi}_1, \ldots \vec{\xi}_{A-2}, \vec{\xi}\,)\; \Phi^*_{\mathrm{H}}(\vec{\xi}_1, \ldots \vec{\xi}_{A-2}, \vec{\xi}\,)\right] \end{aligned}$$

with still unspecified effective momentum $\vec{p}_{eff}$, the momentum transfer $\vec{\Delta} = \vec{q} - \vec{P}_K$, Jacobi coordinates $\vec{\xi}$, and $B = (A-1)/(A-1+m_\Lambda/m_p)$

- OFA is also used to describe the π, p, and $\bar{p}$ scattering off nuclei
- in previous calculations the "frozen-proton" approximation, $\vec{p}_{eff} = 0$ was used as in the Lab frame the amplitude $\mathcal{J}_j(\vec{P}_K, 0, \vec{q}\,)$ has a simple two-component form – only six CGNL amplitudes F_i.

Formalism – the elementary amplitude in a two-component form

- in many-particle calculations the elementary amplitude is rewritten

$$\mathcal{J}\cdot\varepsilon = \overline{u}_\Lambda(\mathrm{p}_\Lambda)\,\gamma_5\left(\sum_{j=1}^{6}\hat{\mathrm{M}}_j\cdot\varepsilon\, A_j(s,t,Q^2)\right)u_\mathrm{p}(\mathrm{p}_\mathrm{p}) = \mathrm{X}_\Lambda^+\,(\vec{J}\cdot\vec{\epsilon})\,\mathrm{X}_\mathrm{p}$$

where $\vec{J}$ has the two-component form and its time component was eliminated using $\epsilon_\mu = \varepsilon_\mu - \varepsilon_0\,\mathrm{q}_\mu/\mathrm{q}_0 = (\,0,\,\vec{\epsilon}\,)$; X_x are Pauli spinors, $s=(\mathrm{q}+\mathrm{p_p})^2$, $t=(\mathrm{q}-\mathrm{P_K})^2$

- a general two-component form has 16 terms

$$\vec{J}\cdot\vec{\epsilon} = G_1\,(\vec{\sigma}\cdot\vec{\epsilon}\,) + G_2\, i(\vec{p}_p\times\vec{q}\cdot\vec{\epsilon}\,) + \ldots\, G_{15}\,(\vec{\sigma}\cdot\vec{P}_K)(\vec{p}_p\cdot\vec{\epsilon}\,) + G_{16}\,(\vec{\sigma}\cdot\vec{P}_K)(\vec{P}_K\cdot\vec{\epsilon}\,)$$

the CGLN-like amplitudes G_i are written via A_j and not all of them are independent

- in the special case: $\vec{p}_p = 0$ (similarly in the c.m. frame, $\vec{p}_p = -\vec{q}$)

$$\begin{aligned}\vec{J}_{LAB}\cdot\vec{\epsilon} \quad = \quad & G_1\,(\vec{\sigma}\cdot\vec{\epsilon}\,) + G_3\, i(\vec{P}_K\times\vec{q}\cdot\vec{\epsilon}\,) + G_8\,(\vec{\sigma}\cdot\vec{q}\,)(\vec{q}\cdot\vec{\epsilon}\,) + \\ & G_{10}\,(\vec{\sigma}\cdot\vec{q}\,)(\vec{P}_K\cdot\vec{\epsilon}\,) + G_{14}\,(\vec{\sigma}\cdot\vec{P}_K)(\vec{q}\cdot\vec{\epsilon}\,) + G_{16}\,(\vec{\sigma}\cdot\vec{P}_K)(\vec{P}_K\cdot\vec{\epsilon}\,)\end{aligned}$$

- the general two-component form was already used by Mart et.al for ${}^3_\Lambda\mathrm{H}$ [Nucl. Phys. A 640, 235 (1998)]

Formalism – the spherical amplitudes

- it is convenient to use the spherical components

$$\vec{J}\cdot\vec{\epsilon} = -\sqrt{3}\,[\,J^{(1)}\otimes\epsilon^{(1)}\,]^0 = \sum_{\lambda}(-1)^{-\lambda}\,J^{(1)}_{\lambda}\epsilon^{(1)}_{-\lambda} = \sum_{\lambda S\xi}\,(-1)^{-\lambda}\,\mathcal{F}^{S}_{\lambda\xi}\,\sigma^{S}_{\xi}\,\epsilon^{(1)}_{-\lambda}$$

where $\mathcal{F}^S_{\lambda\xi}$ are the spherical amplitudes with $S=0,1$ (spin non-flip and flip) and with projections λ (photon), $\xi=\pm1,0$; σ^1_ξ are the spherical Pauli matrixes and $\sigma^0=1$

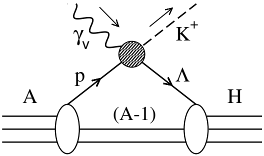
- then explicitly

$$\begin{aligned}\vec{J}\cdot\vec{\epsilon} = \; & -\;\epsilon^{\,1}_{\,1}\,(\,\mathcal{F}^{\,0}_{-10}+\sigma^1_{1}\,\mathcal{F}^{\,1}_{-11}+\sigma^1_0\,\mathcal{F}^{\,1}_{-10}+\sigma^1_{-1}\,\mathcal{F}^{\,1}_{-1-1}\,)\,+\\ & +\;\epsilon^{\,1}_{\,0}\,(\,\mathcal{F}^{0}_{00}\;+\sigma^1_1\,\mathcal{F}^{1}_{01}\;+\sigma^1_0\,\mathcal{F}^{1}_{00}\;+\sigma^1_{-1}\,\mathcal{F}^{\,1}_{0-1}\,)\,-\\ & -\;\epsilon^{\,1}_{-1}\,(\,\mathcal{F}^{0}_{10}\;+\sigma^1_1\,\mathcal{F}^{1}_{11}\;+\sigma^1_0\,\mathcal{F}^{1}_{10}\;+\sigma^1_{-1}\,\mathcal{F}^{\,1}_{1-1}\,)\end{aligned}$$

- the spherical amplitudes $\mathcal{F}^S_{\lambda\xi}$ are expressed via the CGLN-like amplitudes G_i.
- here we will use the elementary amplitudes:
  SLA [Phys. Rev. C **53**, 2613 (1996); **58**, 75 (1998)]
  BS3 [Phys. Rev. C **93**, 025204 (2016); **97**, 025202 (2018)]

Formalism – the optimum on-shell approximation

- the general form of elementary amplitude allows to use "arbitrary" $\vec{p}_{eff}$
- the asumptions:



  - many-body energy-momentum conservation holds $\rightarrow |\vec{\mathrm{P}}_{\mathrm{K}}|_{mb}$
  - binding effects are neglected: $e_{\mathrm{p}} - e_{\Lambda} \approx 0$
  - the elementary amplitude is on-shell: p and Λ are on-mass-shell and $q + p_p = P_K + p_\Lambda$

- these asumptions can only be accomplished with a special "optimum" value of $\vec{p}_{eff}$ where the magnitude of the optimum momentum satisfies

$$E_\gamma + \sqrt{m_p^2 + (\vec{p}_{opt})^2} = \sqrt{m_\Lambda^2 + |\vec{\mathrm{P}}_{\mathrm{K}}|^2_{mb}} + \sqrt{m_\Lambda^2 + (\vec{\Delta} - \vec{p}_{opt})^2}$$

with the momentum transfer $\vec{\Delta} = \vec{\mathrm{q}} - \vec{\mathrm{P}}_{\mathrm{K}}$. $|\vec{p}_{opt}|$ is not unique, it depends on $\theta_{\Delta p}$. Note that in calculations with frosen-proton ($\vec{p}_{eff} = 0$) and on-shell elementary amplitude there are two values of kaon momnetum $|\vec{\mathrm{P}}_{\mathrm{K}}|_{mb} \neq |\vec{\mathrm{P}}_{\mathrm{K}}|_{2b}$.

- one can fix $|\vec{p}_{opt}| = p_{mean} = \sqrt{2\mu\langle T_{kin}\rangle}$ and determine $\theta_{\Delta p}$

Formalism – further details

- partial-wave decomposition: $\mathcal{X}_K^* \, \mathcal{X}_\gamma = \sum F_{LM}(B|\vec{\xi}\,|, \vec{\Delta}) \, Y_{LM}(\hat{\xi})$
  where F_{LM} includes the spherical Bessel function in PWIA
  or a projected kaon distorted wave in DWIA
- the kaon distorted wave function in eikonal approximation with $\mathrm{V}_{\mathrm{opt}}^{\mathrm{1st}}$
  depends on σ_{KN}^{tot}, $\alpha_\circ = \mathrm{Re}\; f^{KN}(0)/\mathrm{Im}\; f^{KN}(0)$, and nucleus density $\rho(\vec{r}\,)$;
  $f^{KN}(0)$ is the isospin averaged forward angle amplitude from
  a separable model with partial waves $\ell = 0, 1, \ldots\, 7$ $(0 \leq \mathrm{E}_{kin} \leq 2\ \mathrm{GeV})$;
- single-particle basis: $|\alpha\rangle = |n\,\ell\,\frac{1}{2}\,j\,\rangle$
- the many-particle structure is included via reduced matrix elements of the single-particle operator (OBDME) $(\,\Psi_H(J_H)\,||\,\left[b_{\alpha'}^{+} \otimes a_\alpha\right]^J\,||\,\Psi_A(J_A)\,)$
  obtained from the shell-model, Tamm-Dancoff,... calculations
- the radial part of the transition operator is given by radial integrals:
  $\mathcal{R}_{\alpha'\alpha}^{LM} = \int_0^\infty dr\, r^2\, R_{\alpha'}^* F_{LM}\, R_\alpha$ where R_α are the radial single-particle wave functions of p and Λ (HO, Woods-Saxon, HF)

Formalism – the differential cross section

- it is suitable to introduce reduced (partial-wave) amplitudes:

$$\mathcal{T}_\lambda^{(1)} = \sum_{Jm} \frac{1}{[J_H]} C^{J_H M_H}_{J_A M_A J m} \, A^\lambda_{Jm}$$

which read as

$$A^\lambda_{Jm} = \frac{1}{[J]} \sum_{S\eta} \mathcal{F}^S_{\lambda\eta} \sum_{LM} C^{Jm}_{LMS\eta} \sum_{\alpha'\alpha} \mathcal{R}^{LM}_{\alpha'\alpha} \, \mathcal{H}^{LSJ}_{\ell' j' \ell j} \, (\Psi_H || \left[b^+_{\alpha'} \otimes a_\alpha \right]^J || \Psi_A)$$

- the transversal response function:

$$\frac{d\sigma_T}{d\Omega_K} = \frac{\beta}{2(2J_A+1)} \sum_{Jm} \frac{1}{2J+1} \left(|A^{+1}_{Jm}|^2 + |A^{-1}_{Jm}|^2 \right)$$

- the longitudinal response function is given by the longitudinal component of the partial-wave amplitude:

$$\frac{d\sigma_L}{d\Omega_K} = \frac{\beta}{2J_A+1} \sum_{Jm} \frac{1}{2J+1} |A^0_{Jm}|^2$$

Results – the proton Fermi motion in $^{12}\mathrm{C}(\gamma_v, \mathrm{K}^+)^{12}_{\Lambda}\mathrm{B}$

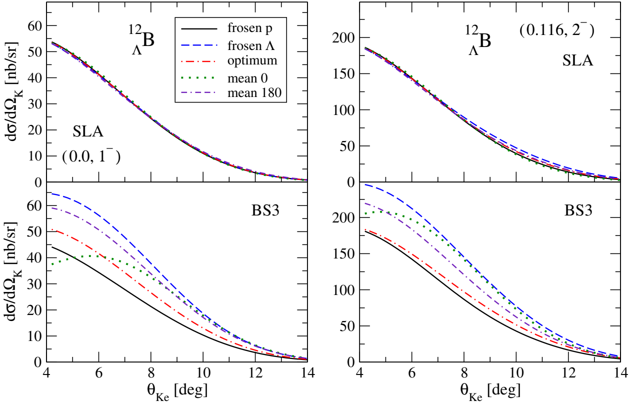
proton momentum: $\vec{p}_{eff} = 0 \quad \Rightarrow \; \vec{p}_\Lambda = \vec{q} - \vec{P}_K = \vec{\Delta}$ (frosen p)

$\vec{p}_{eff} = -\vec{\Delta} \Rightarrow \; \vec{p}_\Lambda = 0$ (frosen Λ)

$\vec{p}_{eff} = \vec{p}_{opt}$ with $\theta_{\Delta p} = 180^\circ$ (optimum)

the mean momentum in ^{12}C: $|\vec{p}_{eff}| = \langle p \rangle = \sqrt{2\mu \langle T_{kin} \rangle}$

$|\vec{p}_{eff}| = 179$ MeV/c ($0p$ orbit) with $\theta_{p\gamma} = 0$ and 180°



kinematics:

$E_\gamma = 2.2$ GeV,

$Q^2 = 0.06$ (GeV/c)^2

$|\vec{\Delta}| \in (240, 350)$ MeV/c

in optimum approximation

$|\vec{p}_{opt}| \in (94, 183)$ MeV/c

$|\vec{p}_\Lambda| \in (166, 265)$ MeV/c

OBDME from shell-model calculations by Millener with dominant transition

$0p_{3/2} \rightarrow 0s_\Lambda$

[P. B. et al, PHYSICAL REVIEW C 108, 024615 (2023)]

Results – the proton Fermi motion effects

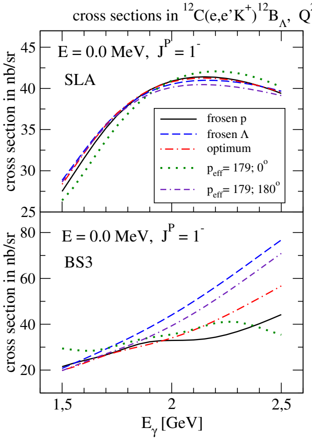
a dependence on E_γ with $Q^2 = 0.06\ (\mathrm{GeV/c})^2$, $\varepsilon = 0.7$, $\Phi_K = 180^\circ$, $\theta_{Ke} = 6^\circ$

– the selection rule: $A^0_{Jm} \sim C^{J_H 0}_{1010} \Rightarrow$ the longitudinal contributions only for even J_H

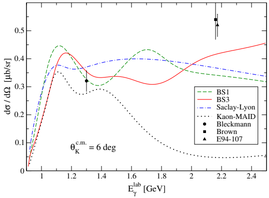
– the longitudinal contributions depend quite strongly on $\vec{p}_{eff}$

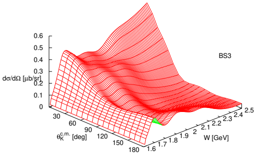
– the effects depend on the elementary amplitude

$\rightarrow$ at $E_\gamma > 2.2$ GeV BS3 predicts rising cross sections



rising cross sections with BS3





[D. Skoupil, P. B.,

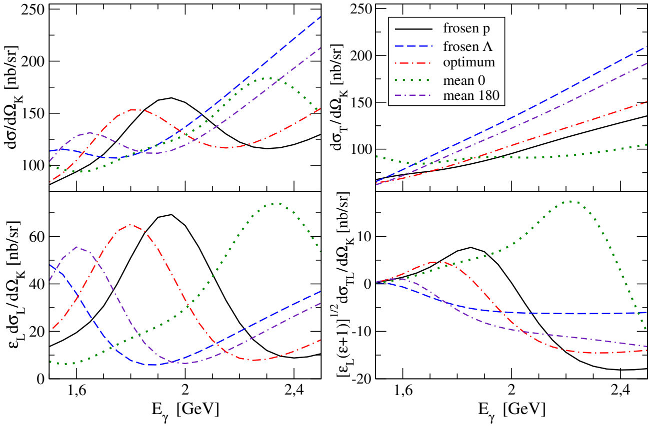
Phys. Rev. C 97 (2018) 025202]

Results – the proton Fermi motion

the separated cross sections for the excited state (11.132, 3^+) of ${}^{12}_{\Lambda}$B

– the longitudinal part of the reduced amplitude contributes: $A^0_{30} \sim C^{30}_{2010} = \sqrt{3/5}$

– $d\sigma_L$ and $d\sigma_{TL}$ are important making the difference between photoproduction ($d\sigma_{\rm T}$) and electroproduction ($d\sigma$) results even at a small photon virtuality, $Q^2 = 0.06$ GeV^2



Fermi motion effects

– very important in $d\sigma_{\rm L}$ and $d\sigma_{\rm TL}$

– depend on $|\vec{p}_{eff}|$ and elementary amplitude

Results – mass effects in the cross sections of $^{208}_{\Lambda}$Tl

– an uncertainty in the ground state hypernucelus mass M_H^0 (the binding energy)

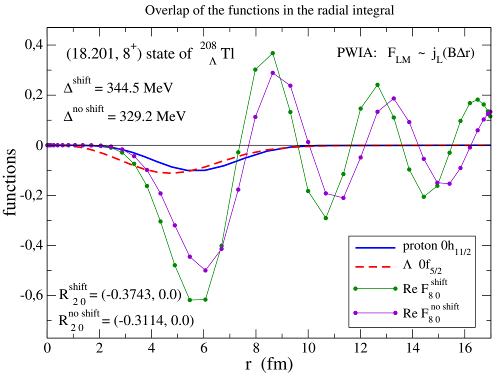
– considering a shift of the hypernucleus mass due to excitation: $M_H^* = M_H^0 + E^*$

$\Rightarrow |\vec{P}_K|, |\vec{\Delta}|, |\vec{p}_{opt}|, \mathcal{F}^S_{\lambda\eta}$ and $\mathcal{R}^{LM}_{\alpha'\alpha}$ depend on E^*

– effects of the mass shift in the states (E^*[MeV], J^P) of $^{208}_{\Lambda}$Tl

(PWIA, TD$_\Lambda$, Nijmegen F YNG, $E_\gamma = 1.5$ GeV, $Q^2 = 0.0383$ GeV^2, $\theta_K = 8^\circ$)

	(11.964, 8^-)	(18.201, 8^+)
M_H	0.006%	0.009%
$\lvert\vec{P}_K\rvert$	-1.0%	-1.6%
$\lvert\vec{\Delta}\rvert$	3.0%	4.6%
$\lvert\vec{p}_{opt}\rvert$	-15.7%	-23.5%
Re$\mathcal{F}^1_{00}$	-1.2%	-4.5%
Im$\mathcal{F}^1_{00}$	-31.4%	-44.4%
$\mathcal{R}^{LM}_{\alpha'\alpha}$	$\mathcal{R}^{70}_{\alpha'\alpha}$: -3.7%	$\mathcal{R}^{80}_{\alpha'\alpha}$: 20.2%
	$0h_{11/2} \rightarrow 0d_{5/2}$	$0h_{11/2} \rightarrow 0f_{5/2}$
dσ	-16.0%	-10.6%
dσ_T	-13.2%	-9.6%



Results – effects from kaon distortion in ${}^{12}_{\Lambda}$B

- comparison of PWIA and DWIA cross sections: diff=100*(PWIA-DWIA)/PWIA in kinematics of the E94-107 JLab experiment, $E_\gamma = 2.21$ GeV, $|\vec{P}_K| = 1.96$ GeV, and $\theta_K = 1.8^\circ$ with the KN scattering at $W_{KN} = 2.2$ GeV and $\sigma^{tot}_{KN} = 17.83$ mb
- differences are 35(higher states)–45(lower states)% similarly in SM and TD$_\Lambda$, except the state (11.542, 1^+) where the leading contributions $0p_{3/2} \to 0p_{1/2}$ and $0p_{3/2} \to 0p_{3/2}$ compose themselves together, opposite to the case of the weakly populated state (10.706, 1^+) where the relative phase is negative (the pahse is given by the sign at the OBDME and the radial integral)

	cross sections in TD$_\Lambda$					cross sections in SM			
energy MeV	J^P	PWIA nb/sr	DWIA nb/sr	diff %	energy MeV	J^P	PWIA nb/sr	DWIA nb/sr	diff %
0.000	1^-	126.97	70.34	45	0.000	1^-	74.14	41.65	44
0.060	2^-	406.68	224.63	45	0.116	2^-	268.38	150.10	44
10.667	2^+	107.46	69.30	36	10.480	2^+	17.11	11.09	35
10.706	1^+	13.11	8.25	37	10.525	1^+	8.89	5.53	38
10.845	2^+	63.27	40.71	36	11.059	2^+	84.63	54.66	35
10.986	3^+	286.20	187.23	35	11.132	3^+	179.72	117.85	34
11.542	1^+	1.17	3.58	-205	11.674	1^+	2.34	4.72	-102

Results – effects from kaon distortion in ${}^{48}_{\Lambda}$K

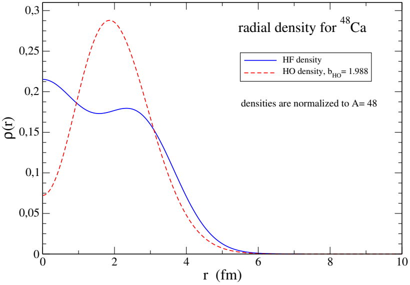
– the kaon-nucleus optical potential

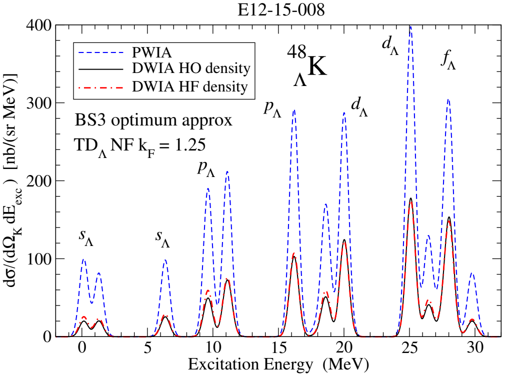
$$V^{1st}_{opt}(\vec{r}\,) = -\beta\,\sigma^{tot}_{KN}(s)\,\frac{1}{2}[\,\mathrm{i} + \alpha(s)\,]\,\rho(\vec{r}\,)$$

– the kaon distorted wave function in eikonal approximation

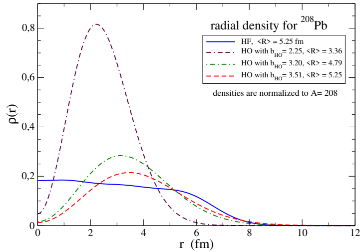
$$\chi^*(\vec{r}\,) = \exp\left[-i\vec{P}_K\cdot\vec{r} - b\,\sigma^{tot}_{KN}\,(1-i\alpha)\int_0^\infty dt\,\rho(\vec{r}+\hat{p}\,t)\right]$$

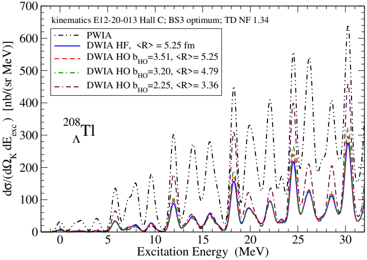
– distortion is 50–60% and depends mainly on behavior of $\rho(\vec{r}\,)$ in the peripheral region





Results – effects from kaon distortion in $^{208}_{\Lambda}$Tl are ≈ 50–90%





cross sections in nb/sr

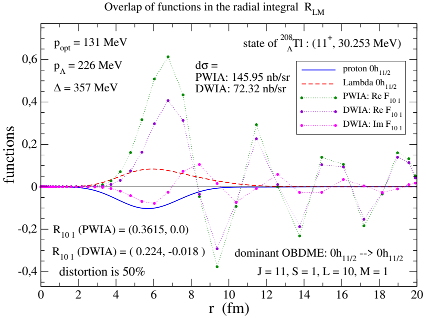
E^*, J^P	l_Λ	PWIA	DWIA HO	diff %	DWIA HF	diff %
0.000, 6^-	s_Λ	11.96	1.88	84	2.07	82
3.857, 3^+	s_Λ	12.67	1.17	90	1.26	90
5.794, 6^+	p_Λ	25.34	5.13	79	5.05	80
5.804, 7^+	p_Λ	52.81	11.73	77	11.69	77
7.955, 2^-	p_Λ	27.72	3.15	88	3.51	87
7.969, 3^-	p_Λ	34.79	4.44	87	5.00	85
11.964, 8^-	d_Λ	111.26	33.27	70	30.93	72
13.705, 4^-	p_Λ	22.49	2.75	87	3.11	86
13.720, 5^-	p_Λ	24.36	3.82	84	4.10	83
15.692, 5^+	d_Λ	91.79	14.42	84	16.12	82
18.201, 8^+	f_Λ	86.00	28.09	67	25.52	70
20.286, 3^+	p_Λ	11.01	1.01	90	1.07	90
21.685, 2^-	p_Λ	42.57	7.24	82	7.72	81
22.034, 6^-	f_Λ	81.52	17.95	77	18.87	76
24.453, 7^-	g_Λ	28.05	7.25	74	6.59	76
24.472, 9^-	g_Λ	92.66	36.96	60	33.19	64
26.220, 7^-	f_Λ	78.46	20.99	73	19.94	74
28.673, 8^-	f_Λ	106.61	27.27	74	25.85	75
29.343, 5^+	d_Λ	73.59	14.43	80	15.99	78
30.246, 9^+	h_Λ	36.21	12.56	65	10.86	70
30.253, 11^+	h_Λ	147.78	79.76	46	73.71	50
32.361, 7^+	g_Λ	101.38	29.12	71	26.52	73
32.443, 8^+	g_Λ	81.60	28.02	65	25.54	68

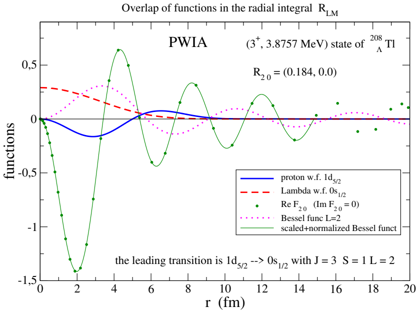
Results – kaon distortion

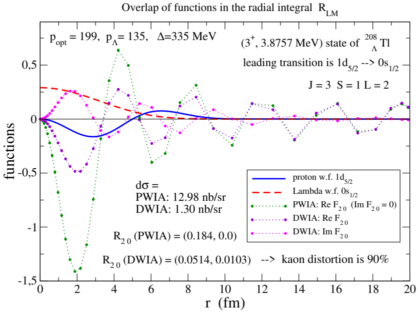
the different effects are due to the radial integral:

$$\mathcal{R}^{LM}_{\alpha'\alpha} = \int_0^\infty dr\, r^2\, R^*_{\alpha'} F_{LM}\, R_\alpha$$

real in PWIA and complex in DWIA







Results – effects from the nucleus-hypernucleus structure

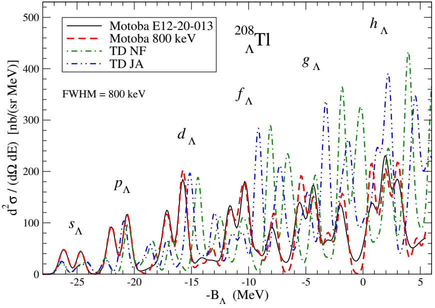
– Tamm-Dancoff self-consistent many-body calculations using the NN Daejeon16 interaction with a phenomenological density dependent (DD) term and effective YN interactions;

– the effective YN interactions: Nijmegen-F and Jülich A YNG with $k_F = 1.34$ fm

– kinematics: $E_\gamma = 1.5$ GeV, $Q^2 = 0.012$ GeV^2, $|\vec{P}_K| = 1.176$ GeV, $\theta_K = 0.5°$

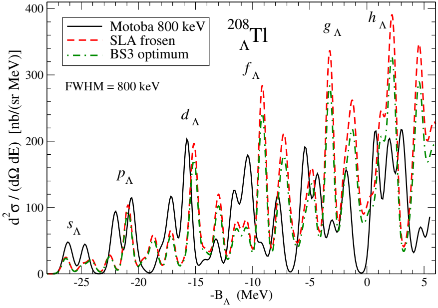
– the rising background in TD$_\Lambda$ is due to many weakly populated states

– the result by Motoba and Millener are from the proposal of the JLab experiment E12-20-013



different YNG interactions

with the SLA amplitude



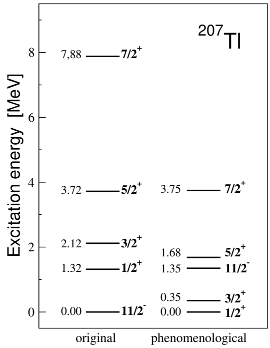
different elementary amplitudes

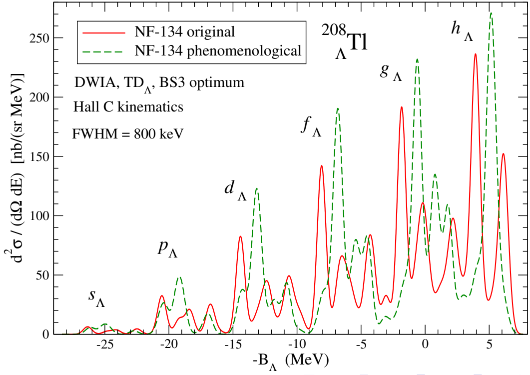
with the Jülich A YNG interaction

Results – tuning the nucleus structure

- nucleus is described as a core-particle system: $|^{208}\mathrm{Pb}_{g.s.}\rangle = \sum |^{207}\mathrm{Tl}\,(E_c,\, J_c^{P_c})\rangle \otimes |(n\,l\,j)_p\rangle$
- similarly hypernucleus: $|^{208}_{\Lambda}\mathrm{Tl}\,(E_H,\, J_H^{P_H})\rangle = \sum |^{207}\mathrm{Tl}\,(E_c,\, J_c^{P_c})\rangle \otimes |(n'\,l'\,j')_\Lambda\rangle$
- the core nucleus is a "spectator" in the impulse approximation
- spectrum of the core nucleus calcualted in TD_Λ with the NN Daejeon16 interaction and the DD term (original) is modified introducing phenomenological energies

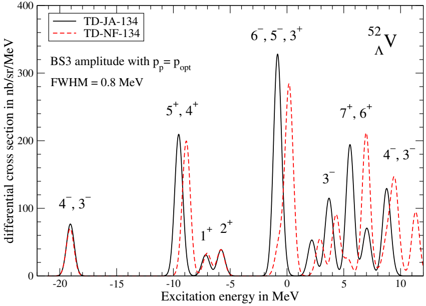
  $\longrightarrow$ modification of the $^{208}_{\Lambda}$Tl excitation spectrum

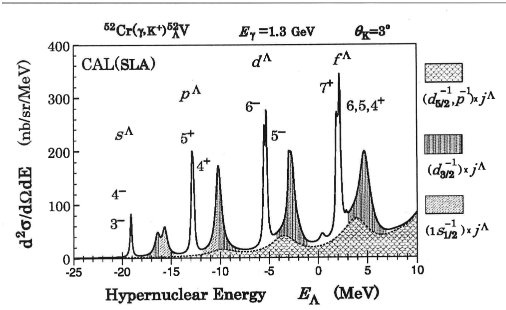




Results – electroproduction of ${}^{52}_{\Lambda}$V

- the excitation spectrum in ^{52}Cr(e,e′K$^+$)${}^{52}_{\Lambda}$V
- the NN Daejeon16 interaction with the phenomenological density dependent term
- the Nijmegen-F and Jülich A YNG interaction with $k_F = 1.34$ fm
- the BS3 amplitude in the optimum on-shell approximation
- kinematics: $E_\gamma = 1.3$ GeV, $Q^2 = 0.003$ GeV^2, and $\theta_K = 3^\circ$
- comparison with older results from P.B., M.Sotona, T.Motoba, K.Itonaga, K.Ogawa, and O.Hashimoto, Nuc. Phys. **881**, 199 (2012).





Results – predictions for the E12-20-013 experiment at JLab

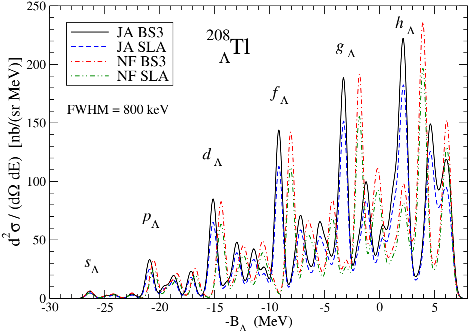
– the rection: $^{208}\mathrm{Pb}(e,e'K^+)^{208}_{\Lambda}\mathrm{Tl}$

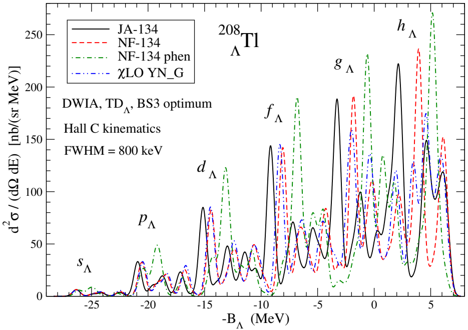
– the many-particle calculation: the Tamm-Dancoff Λ-nucleon particle-hole model

– the NN interaction: Daejeon16 with the phenomenological density dependent term

– the YNG interaction: Nijmegen-F and Jülich A with $k_F = 1.34$ fm

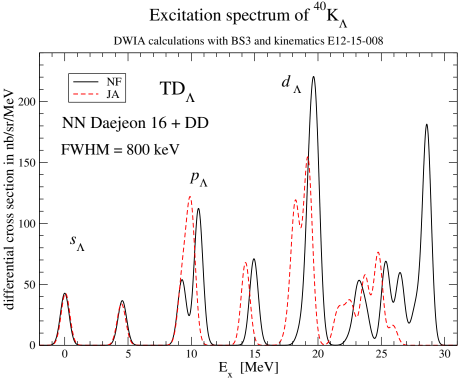
– Hall C kinematics: $E_\gamma = 1.5$ GeV, $Q^2 = 0.0323$ GeV^2, $|\vec{P}_K| = 1.245$ GeV, $\theta_K = 7.1°$, $\Phi_K = 180°$

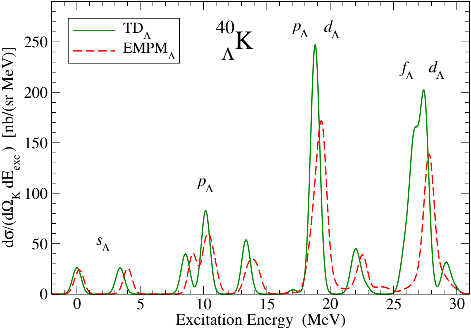




Results – predictions for the E12-15-008 experiment at JLab

- the rection: $^{40}\mathrm{Ca}(\mathrm{e,e'K^+})^{40}_{\Lambda}\mathrm{K}$
- the many-particle calculation: the Tamm-Dancoff Λ-nucleon particle-hole model
- the NN interaction: chiral NNLO$_{sat}$ NN+NNN as in Phys.Rev.C **108**, 024615 (2023)
  and Daejeon 16 with the phenomenological density dependent term
- the YNG interaction: Nijmegen-F and Jülich A with $k_F = 1.30$ fm
- kinematics: $E_\gamma = 1.5$ GeV, $Q^2 = 0.0323$ GeV^2, $|\vec{P}_\mathrm{K}| = 1.245$ GeV, $\theta_\mathrm{K} = 7.1°$, $\Phi_\mathrm{K} = 180°$
- BS3 amplitude in the optimum on-shell approximation

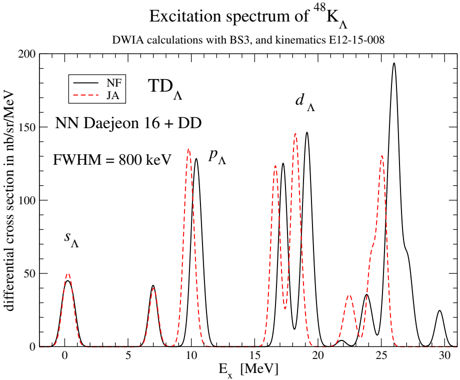


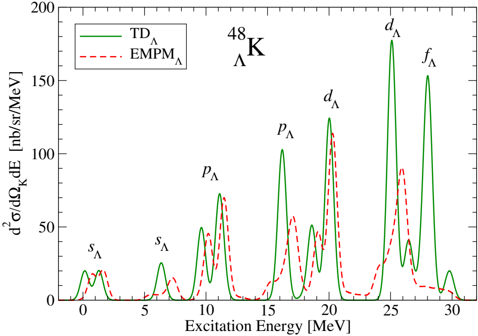


Phys. Rev. C **108**, 024615 (2023)

Results – predictions for the E12-15-008 experiment at JLab

- the rection: $^{48}\text{Ca}(\text{e},\text{e}'\text{K}^+)^{48}_{\Lambda}\text{K}$
- the many-particle calculation: the Tamm-Dancoff Λ-nucleon particle-hole model
- the NN interaction: chiral NNLO$_{sat}$ NN+NNN as in Phys.Rev.C **108**, 024615 (2023)
  and Daejeon 16 with the phenomenological density dependent term
- the YNG interaction: Nijmegen-F and Jülich A with $k_F = 1.34$ fm
- kinematics: $E_\gamma = 1.5$ GeV, $Q^2 = 0.0323$ GeV^2, $|\vec{P}_\text{K}| = 1.245$ GeV, $\theta_\text{K} = 7.1^\circ$, $\Phi_\text{K} = 180^\circ$





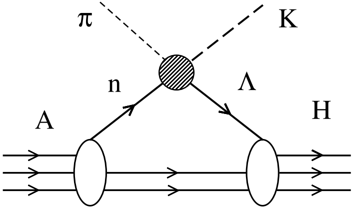
Phys. Rev. C **108**, 024615 (2023)

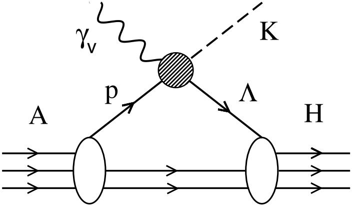
Summary

- the effects from proton Fermi motion in electroproduction of hypernuclei depend on the elementary amplitude and kinematics and they are more apparent in the longitudinal parts of the cross section
- we have suggested using the optimum on-shell approximation that effectively accounts for the proton motion and improves agreement with the data
- the shell-model approach with the effective YN interaction fitted to very precise data from the γ-ray spectroscopy provide reasonable description of spectra in electroproduction of p-shell (light) hypernuclei
- Tamm-Dancoff formalism with the chiral NNLO$_{sat}$ NN+NNN potential and Daejeon 16 with the phenomenological DD term and various effective YN interactions (Nijmegen, Jülich) turn out to be a good tool in analysis of the data in electroproduction of medium ($^{40}_{\Lambda}$K) and heavy ($^{208}_{\Lambda}$Tl) mass hypernuclei
- we provided predictions for the experiments in preparation at JLab
- correct inclusion of kaon distortion is important as in given kinematics the distortion amounts to $\approx$ 30–40% for light (p-shell) and $\approx$ 50–90% for heavy hypernuclei

Outlook

- include Coulomb interaction in the kaon FSI (Z = 81 for ${}^{208}_{\Lambda}$Tl)
- a more ambitious project: develop analogous formalism for (π^+, K^+) production and utilize consistent calculations of the many-particle part (OBDME)





Thank you for your attention!

Backup slide – factorization

- production amplitude in PWIA

$$\mathcal{T}_j = \mathrm{Z}\int d^3\xi_\Lambda\, d^3\xi_p\, e^{iB\vec{\Delta}\cdot\vec{\xi}_\Lambda}\ \mathrm{Tr}\int \frac{d^3 p_p}{(2\pi)^3} \exp\left[i\vec{p}_p\cdot(\vec{\xi}_\Lambda - \vec{\xi}_p)\right] \mathcal{J}_j(\vec{P}_K, \vec{P}_\gamma, \vec{p}_p)$$
$$\int d^3\xi_1 \ldots d^3\,\xi_{A-2}\ \Phi_{\mathrm{A}}(\vec{\xi}_1, \ldots \vec{\xi}_{A-2}, \vec{\xi}_p)\ \Phi^*_{\mathrm{H}}(\vec{\xi}_1, \ldots \vec{\xi}_{A-2}, \vec{\xi}_\Lambda)\,,$$

with the momentum transfer $\vec{\Delta} = \vec{P}_\gamma - \vec{P}_K$, Jacobi coordinates $\vec{\xi}$, a scaling parameter B and assuming translational invariance

$$\langle \vec{P}_K\, \vec{p}_\Lambda \,|\, \hat{\mathrm{J}}_j \,|\, \vec{P}_\gamma\, \vec{p}_p \,\rangle = (2\pi)^3\, \delta^{(3)}(\vec{P}_K + \vec{p}_\Lambda - \vec{P}_\gamma - \vec{p}_p)\, \mathcal{J}_j(\vec{P}_K, \vec{P}_\gamma, \vec{p}_p).$$

– the amplitude is still averaged over the proton momentum $\vec{p}_p$.
– the elementary amplitude $\mathcal{J}_j$ is a matrix 2×2
(in two-component CGLN-like formalism)

- elementary amplitude is factorized out of the integral replacing $\mathcal{J}_\mu(\vec{P}_K, \vec{P}_\gamma, \vec{p}_p) \rightarrow \mathcal{J}_\mu(\vec{P}_K, \vec{P}_\gamma, \vec{p}_{eff})$ for some (unknown) effective proton momentum $\vec{p}_{eff}$.

Backup slide – effective proton momentum

- another possibility how to fix p_{eff} is to use a mean value calculated from the mean kinetic energy of the proton in nucleus

$$\langle T_{kin}\rangle = \frac{\langle p\rangle^2}{2\mu} \Rightarrow \langle p\rangle = \sqrt{2\mu\langle T_{kin}\rangle} = |\vec{p}_{eff}| \quad \text{where}$$

$$\begin{aligned}\langle\Psi_{Eljm}|T_{kin}|\Psi_{Eljm}\rangle &= \int dr\, u^*_{Elj}(r)\left(-\frac{(\hbar c)^2}{2\mu}\right)\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right] u_{Elj}(r) \\ &= \int dr\, |u_{Elj}(r)|^2\, (E - \langle ljm|V|ljm\rangle)\end{aligned}$$

with the Woods-Saxon and Coulomb potentials: $V = WS_c + WS_{LS} + V_{coul}$

and the single-particle state: $\Psi_{Eljm}(\vec{r}\,) = \frac{u_{Elj}(r)}{r}\,[\,\mathrm{Y}_l(\hat{r}) \otimes \chi^{(\frac{1}{2})}\,]^j_m$

- for the ^{12}C target we get results:

proton state	E[MeV]	WS_c	WS_{LS}	$\langle T\rangle$	$\langle p\rangle$
$p_{1/2}$	-10.37	-54.18	18.98	18.16	**176.7**
$p_{3/2}$	-15.96	-54.18	-9.49	18.76	**179.6**

Backup slide – off-shell effects