

Radiation Correction Introduction

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Inclusive electron scattering

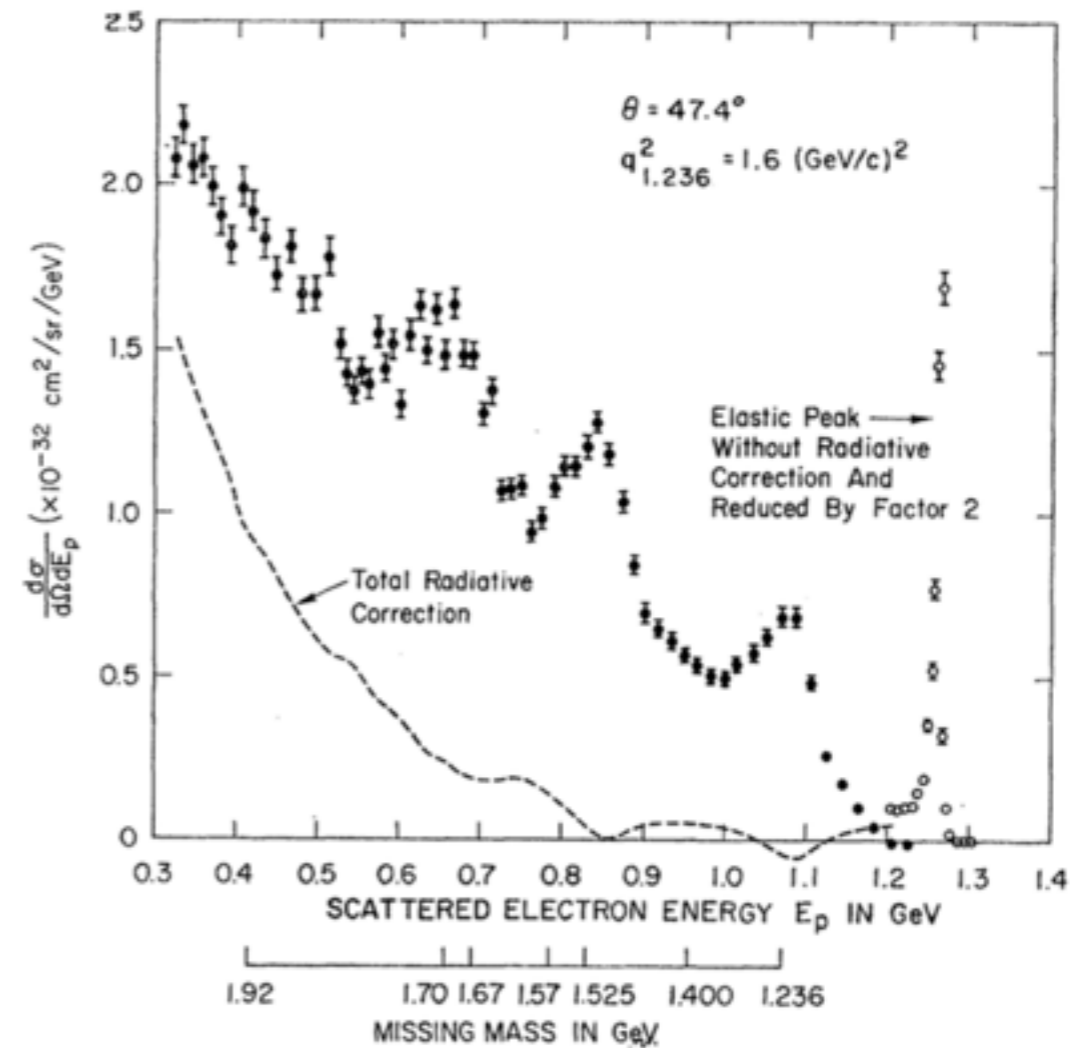
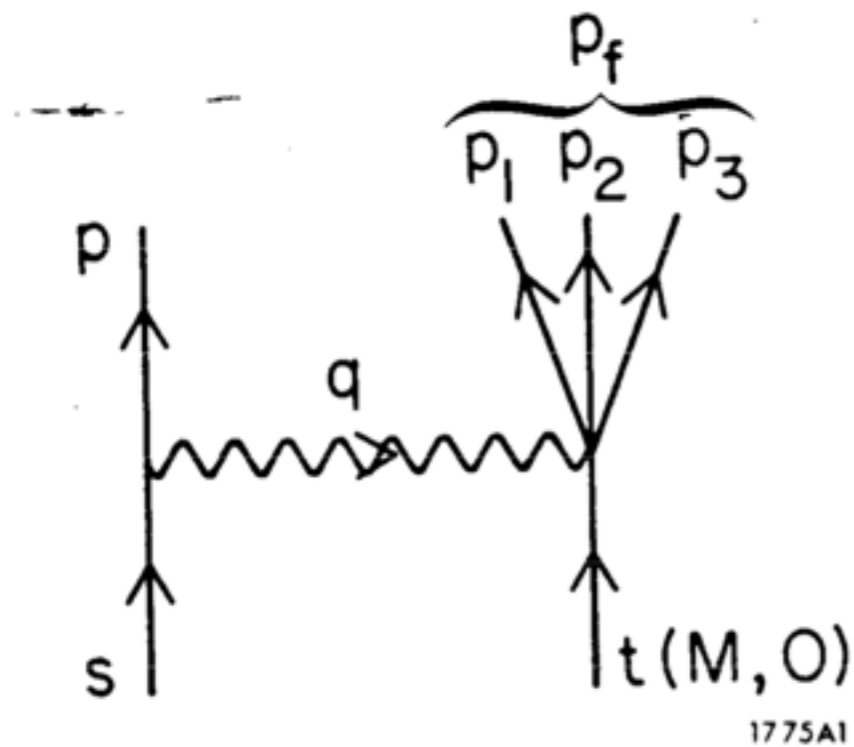


FIG. 1. A typical spectrum of inelastic ep scattering and the radiative corrections. Both of these curves are taken directly from Brasse *et al.* (Ref. 4).

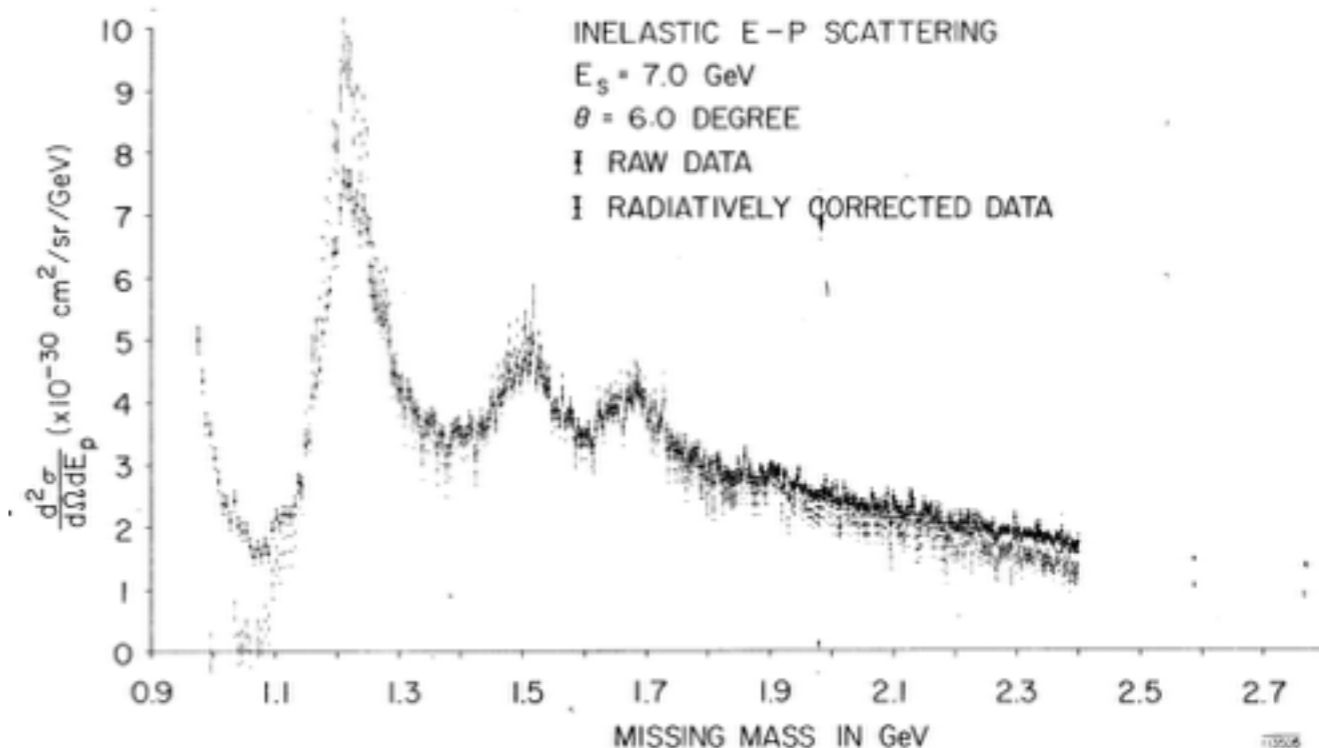
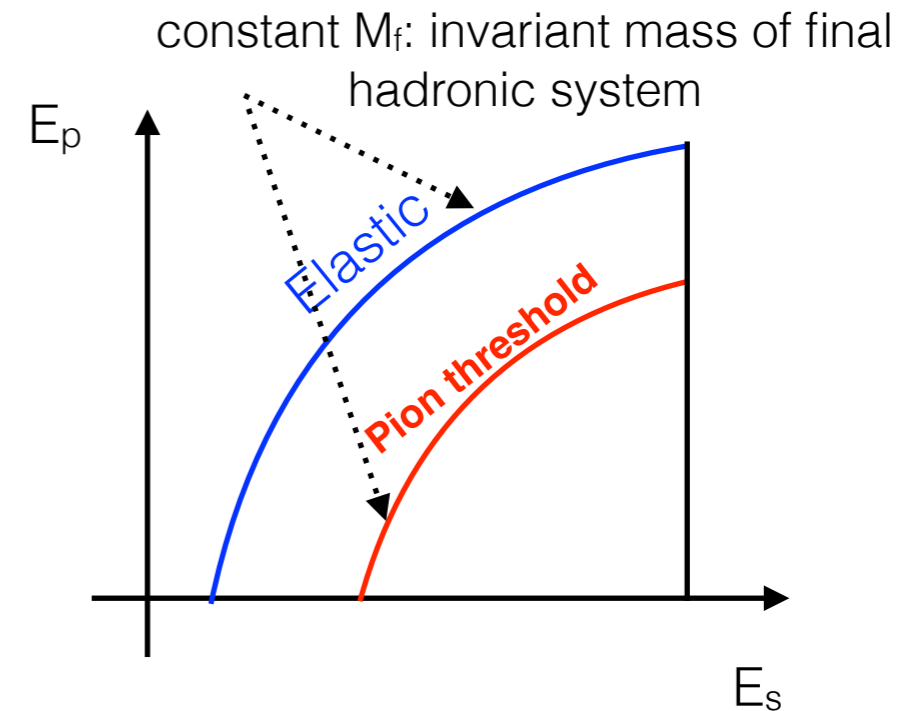
Radiation correction:

1. **external**: straggling effect due to ionization and bremsstrahlung(external) when electrons pass through the medium before and after the scattering;
2. **internal**: bremsstrahlung during the collisions and virtual photons;

Inclusive electron scattering

For multi-GeV electron scattering, there are only about three or four broad bumps. These bumps are above the pion threshold, and hence lie on top of the continuum spectra.

Only the elastic peak needs a special treatment, and all the resonances can be treated the same way as treat the continuum.



Internal radiative correction

1. elastic peak radiative correction

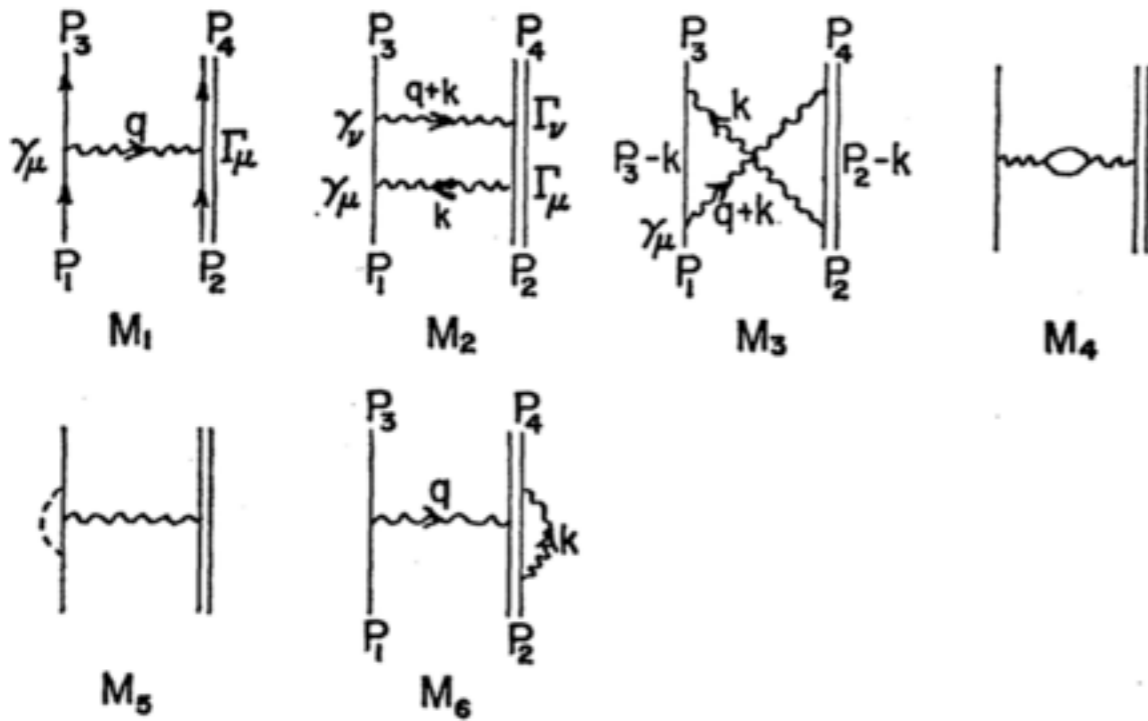


FIG. 2. Feynman diagrams for elastic scattering.

$$d\sigma_{\text{elastic}} = (2\pi)^2 \frac{E_1 E_2}{[(p_1 p_2)^2 - m^2 M^2]^{\frac{1}{2}}}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rosenbluth}}$$

$$\times \frac{1}{4} \int \delta(p_3 + p_4 - p_1 - p_2) d^3 p_3 d^3 p_4$$

$$\times \sum_{\text{spin}} [M_1^\dagger M_1 + \sum_{i=2}^6 2 \text{Re}(M_1^\dagger M_i)]$$

$$\delta_{el} \left(\frac{d\sigma}{d\Omega} \right)_{\text{Rosenbluth}}$$

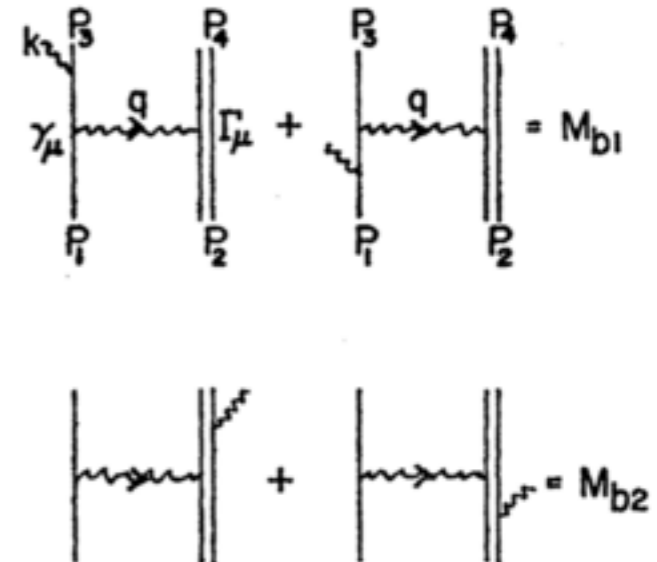


FIG. 3. Feynman diagrams for inelastic scattering.

$$\delta_b \left(\frac{d\sigma}{d\Omega} \right)_{\text{Rosenbluth}}$$

$$\frac{d\sigma_{\text{exp}}}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{rosenbluth}} (1 + \delta_{el} + \delta_b) = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Rosenbluth}} (1 + \delta)$$

[1]. Y.S. Tsai, Phys. Rev. 122, 1898 (1961);

[2]. L. C. Maximon, J. A. Tjon, Phys. Rev. C 62, 054320

Internal radiative correction

2. radiative tail

p_f — four momentum of the final hadron system;

k — four momentum of the photon emitted;

missing mass: $u^2 = (p_f + k)^2 \geq p_f^2 \equiv M_f^2$



the radiative tail from the lighter M_f can affect the measurement of heavier M_f , but not vice versa.

radiative correction for inelastic:

- 1). subtract the elastic radiative tail;
- 2). calculate the radiative tail to the continuum spectrum;

Internal radiative correction

1). radiative tail from elastic scattering

a). Exact formula:

$$\frac{d^2\sigma_r}{d\Omega dp} = \frac{\alpha^3 E_p}{(2\pi)^2 M E_s} \int_{-1}^1 \frac{\omega d(\cos\theta_k)}{2q^4 (u_0 - |u| \cos\theta_k)} \int_0^{2\pi} B_{\mu\nu} T_{\mu\nu} d\phi_l$$

contains elastic form factors

$$\sigma_{\text{ex}} \equiv \left(\frac{d^2\sigma}{d\Omega dE_p} \right)_{\text{ex}} = \frac{\alpha^3}{(2\pi)} \left(\frac{E_p}{E_s} \right) \int_{-1}^1 \frac{2M_T \omega d(\cos\theta_k)}{q^4 (u_0 - |\vec{u}| \cos\theta_k)}$$

$$\begin{aligned} & \times \left(\bar{W}_2(q^2) \left\{ \frac{-am^2}{x^3} \left[2E_s(E_p + \omega) + \frac{q^2}{2} \right] - \frac{a'm^2}{y^3} \left[2E_p(E_s - \omega) + \frac{q^2}{2} \right] \right. \right. \\ & \quad - 2 + 2\nu(x^{-1} - y^{-1}) \{ m^2(s \cdot p - \omega^2) + (s \cdot p)[2E_s E_p - (s \cdot p) + \omega(E_s - E_p)] \} \\ & \quad + x^{-1} \left[2(E_s E_p + E_s \omega + E_p^2) + \frac{q^2}{2} - (s \cdot p) - m^2 \right] \\ & \quad \left. \left. - y^{-1} \left[2(E_s E_p - E_p \omega + E_s^2) + \frac{q^2}{2} - (s \cdot p) - m^2 \right] \right\} \right. \\ & \quad \left. + \bar{W}_1(q^2) \left[\left(\frac{a}{x^3} + \frac{a'}{y^3} \right) m^2(2m^2 + q^2) + 4 + 4\nu(x^{-1} - y^{-1})(s \cdot p)(s \cdot p - 2m^2) \right. \right. \\ & \quad \left. \left. + (x^{-1} - y^{-1})(2s \cdot p + 2m^2 - q^2) \right] \right) , \end{aligned} \tag{A24}$$

Internal radiative correction

1). radiative tail from elastic scattering

b). angle-peaking approximation

L. W. MO and T. S. Tsai, Rev. Mod. Phys. 41, 205 (1969), Appendix C;

$$\begin{aligned}\sigma_p &\equiv \left(\frac{d^2\sigma}{d\Omega dE_p} \right)_{\text{peak approx.}} \\ &= \frac{M_T + 2(E_s - \omega_s) \sin^2(\frac{1}{2}\theta)}{M_T - 2E_p \sin^2(\frac{1}{2}\theta)} \bar{\sigma}_{\text{el}}(E_s - \omega_s) \left[\frac{bt_r \phi(v_s)}{\omega_s} \right] \\ &\quad + \bar{\sigma}_{\text{el}}(E_s) \left[\frac{bt_r \phi(v_p)}{\omega_p} \right],\end{aligned}\tag{A56}$$

Phys. Rev. D 12, 1884 (1975), Appendix A

It saves a lot of time. But when $E_p < \frac{1}{3} E_{p\text{max}}$, the peaking approximation can be in error as much as 30 to 40%. Exact formula must be used when energy loss is large.

Internal radiative correction

2). radiative tail for continuum state

After subtracting the elastic tail from the inelastic spectrum, the radiative correction to the continuum spectrum can be calculated by integrating with respect to M_f^2 from pion threshold

$$(d\sigma_r/d\Omega dp)(E_s, E_p) = (d\sigma/d\Omega dp)(E_s, E_p)[1 + \delta_r(\Delta)] + (d\sigma_r/d\Omega dp)(w > \Delta),$$

$$\frac{d^2\sigma_r}{d\Omega dp}(w > \Delta) = \frac{\alpha^3 E_p}{(2\pi)^2 M E_s} \int_{-1}^1 d(\cos\theta_k) \int_{\Delta}^{\omega_{\max}(\cos\theta_k)} \frac{\omega d\omega}{2q^4} \int_0^{2\pi} B_{\mu\nu}^c T_{\mu\nu} d\phi_k$$

$$\omega = \frac{1}{2}(u^2 - M_f^2) / (u_0 - |u| \cos\theta_k)$$

contains structure functions

$$F(q^2, M_f^2), G(q^2, M_f^2)$$

It's impossible to know $F(q^2, M_f^2), G(q^2, M_f^2)$ for all continuum state, so angle-peaking approximation is used to derive an approximation expression.

L. W. MO and T. S. Tsai, Rev. Mod. Phys. 41, 205 (1969), Appendix C;

Internal radiative correction

3). Equivalent radiators method

The effect of the internal bremsstrahlung on the elastic or inelastic scattering is equivalent to placing one radiator before the scattering and another radiator of the same thickness after the scattering.

The thickness of each radiator is equal to:

$$t_r = b^{-1}(\alpha/\pi) [\ln (2sp/m^2) - 1],$$

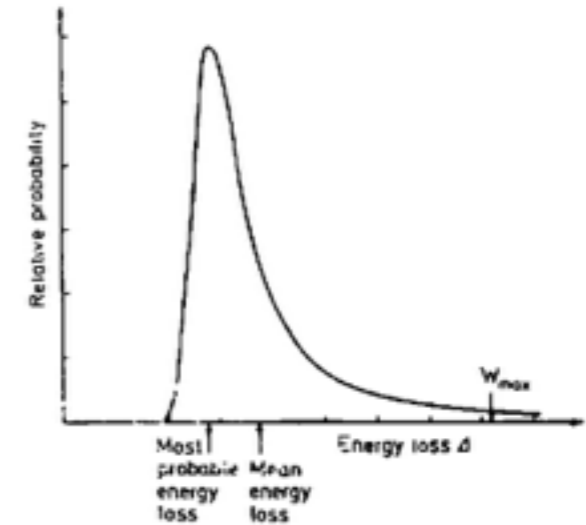
The equivalent radiator can be added to T(target length), when doing straggling effect correction.

L. W. MO and T. S. Tsai, *Rev. Mod. Phys.* 41, 205 (1969), part III compare these two approximations with the exact formula. Basically, both approximations work well near the peak. Since the radiative tails from an inelastic event affect only its immediate neighborhood, the approximation works very well.

External radiative correction

Straggling effect

When passing through the target, electrons will lose energy due to ionization and bremsstrahlung. The amount of energy loss is not equal to the mean value but has statistical fluctuations.



$I(E_0, E, t)dE$ ——— the probability of finding an electron in the energy interval between E and $E+dE$ at a depth t (in units of radiation length)



measured cross section due to straggling effect:

$$\sigma_{\text{exp}}(E_s, E_p) = \int_0^T \frac{dt}{T} \int_{E_{s \min}(E_p)}^{E_s} dE'_s \int_{E_p}^{E_{p \max}(E'_s)} dE'_p I(E_s, E'_s, t)$$

$$\sigma_r(E'_s, E'_p) I(E'_p, E_p, T-t) ,$$

cross section including internal radiative correction

Straggling effect

By energy peaking approximation:

$$\sigma_{\text{exp}}(E_s, E_p) = \int_0^T \frac{dt}{T} \int_{E_{s \min}(E_p)}^{E_s} dE'_s \int_{E_p}^{E_{p \max}(E'_s)} dE'_p I(E_s, E'_s, t) \sigma_r(E'_s, E'_p) I(E'_p, E_p, T-t)$$



only consider bremsstrahlung,
ionization will be included later

1). radiative cross section for discrete level:

$$\sigma_b^j(E_s, E_p) \equiv \int_0^1 \frac{dt}{T} \int_{E_{s \min}(E_s)}^{E_s} dE'_s \int_{E_p}^{E_{p \max}(E'_s)} dE'_p \left[I_b(E_s, E'_s, t) \frac{d\sigma_j(E'_s)}{d\Omega} \right. \\ \left. \frac{1}{2M+2E'_s(1-\cos\theta)} \delta(M^2 - M_j^2 + 2M(E'_s - E'_p) - 2E'_s E'_p(1-\cos\theta)) I_b(E'_p, E_p, T-t) \right] \quad (\text{C.9})$$

$$\approx \int_0^T \frac{dt}{T} \left[\frac{d\sigma_j(E_s)}{d\Omega} I_b(E_p + \omega_p, E_p, T-t) \int_{E_s - \omega_s}^{E_s} dE'_s I_b(E_s, E'_s, t) \right. \\ \left. + \frac{d\sigma_j(E_s - \omega_s)}{d\Omega} \frac{M + (E_s - \omega_s)(1-\cos\theta)}{M - E_p(1-\cos\theta)} I_b(E_s, E_s - \omega_s, t) \right. \\ \left. \times \int_{E_p}^{E_p + \omega_p} dE'_p I_b(E'_p, E_p, T-t) \right] \quad (\text{C.10})$$

can be calculated from Moller cross section

T. S. Tsai, SLAC-PUB-848 (1971), Appendix B;

Straggling effect

1). radiative cross section for discrete level:

The integration with respect to t can be approximated by assuming that the scattering took place exactly at $t=T/2$:

$$\sigma_b^j(E_s, E_p) \approx (1 + 0.5772 bT) \left[\frac{d\sigma_j(E_s)}{d\Omega} \frac{bT}{2} \frac{1}{\omega_p} \phi\left(\frac{\omega_p}{E_p + \omega_p}\right) + \frac{d\sigma_j(E_s - \omega_s)}{d\Omega} \frac{M + (E_s - \omega_s)(1 - \cos\theta)}{-M - E_p(1 - \cos\theta)} \frac{bT}{2} \frac{1}{\omega_s} \phi\left(\frac{\omega_s}{E_s}\right) \right] \left(\frac{\omega_s}{E_s}\right)^{bT/2} \left(\frac{\omega_p}{E_p + \omega_p}\right)^{bT/2}$$

1

The half length approximation is a very good approximation when $bT < 0.1$

$$b = \frac{4}{3} \left\{ 1 + \frac{1}{9} \left[\frac{(Z+1)}{(Z+\eta)} \right] \left[\ln(183 Z^{-1/3}) \right]^{-1} \right\}$$

The target initial window and final window thickness will be added separately to $T/2$ finally

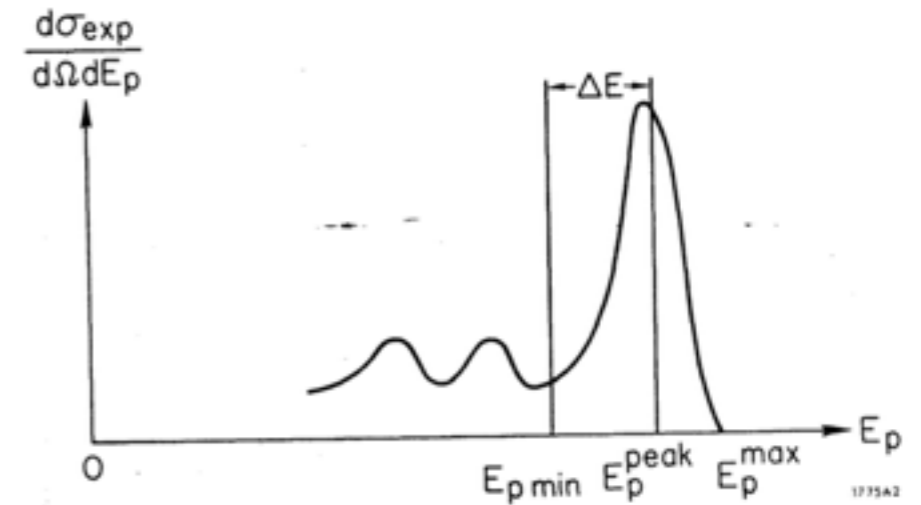
Straggling effect

2). radiative cross section for elastic peak:

$$\int_{E_{p \max} - \Delta E}^{E_{p \max}} \sigma_b^j(E_s, E_p) dE_p = (1 + 0.5772 bT) \left(\frac{R\Delta E}{E_s} \right)^{bT/2} \left(\frac{\Delta E}{E_{p \max}} \right)^{bT/2} \frac{d\sigma_j(E_s)}{d\Omega} \quad (\text{C.20})$$

2

T. S. Tsai, SLAC-PUB-848 (1971), Appendix C;



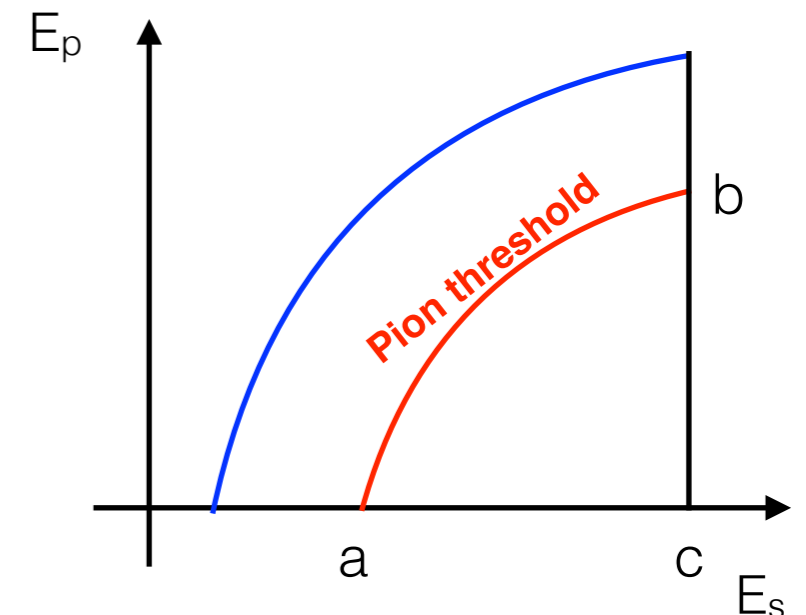
3). radiative cross section for continuum state:

Radiative cross section for continuum state can be regarded as a sum of many discrete states. It's calculated by integrating ① with respect to M_f^2 from pion threshold along lines a to c and b to c:

$$\begin{aligned} \sigma_b^c(E_s, E_p) = & (1 + 0.5772 bT) \left[\left(\frac{R\Delta}{E_s} \right)^{bT/2} \left(\frac{\Delta}{E_p} \right)^{bT/2} \sigma(E_s, E_p) \right. \\ & + \int_{E_p + \Delta}^{E_{p \max}(E_s)} dE'_p \sigma(E_s, E'_p) \left(\frac{E'_p - E_p}{E'_p} \right)^{bT/2} \left(\frac{E'_p - E_p}{E_s} \right)^{bT/2} \frac{bT}{2(E'_p - E_p)} \phi \left(\frac{E'_p - E_p}{E'_p} \right) \\ & \left. + \int_{E_{s \min}(E_p)}^{E_s - R\Delta} dE'_s \sigma(E'_s, E_p) \left(\frac{E_s - E'_s}{E_p R} \right)^{bT/2} \left(\frac{E_s - E'_s}{E_s} \right)^{bT/2} \frac{bT}{2(E_s - E'_s)} \phi \left(\frac{E_s - E'_s}{E_s} \right) \right] \quad (\text{C.23}) \end{aligned}$$

3

T. S. Tsai, SLAC-PUB-848 (1971), Appendix C;



Radiative cross section

1).elastic peak:

After including the effects due to the ionization and the internal radiative correction to the radiative cross section for elastic peak is:

2

$$\frac{d\sigma_{\text{exp}}}{d\Omega}(\Delta E) = G(q^2, T) \frac{d\sigma}{d\Omega} e^{\delta} \left(\frac{\Delta ER}{E_s}\right)^{bT/2} \left(\frac{\Delta E}{E_p^{\text{peak}}}\right)^{bT/2} \times \left(1 - \frac{\xi}{\Delta E}\right)$$

T. S. Tsai, SLAC-PUB-848 (1971), Chp 2

$$G(q^2, T) = 1 + 0.5772bT + \frac{\alpha}{\pi} \left[\frac{1}{6} \pi^2 - \Phi(\cos^2 \frac{\theta}{2}) \right].$$

$$\int_{E_{p\text{max}} - \Delta E}^{E_{p\text{max}}} \sigma_b^j(E_s, E_p) dE_p = (1 + 0.5772bT) \left(\frac{R\Delta E}{E_s}\right)^{bT/2} \left(\frac{\Delta E}{E_{p\text{max}}}\right)^{bT/2} \frac{d\sigma_j(E_s)}{d\Omega} \quad \text{(C.20)}$$

2

2). radiative tail from elastic peak:

After including the effects due to the ionization and the internal radiative correction to the radiative tail from elastic scattering can be obtained.

1

Since the internal bremsstrahlung can be exact or approximate, there are two formulas can be used for elastic tail, which can be found in [Phys. Rev. D 12, 1884 \(1975\), Appendix A](#)

Radiative cross section

3). continuum spectrum (elastic tail is subtracted)

- For the internal radiative cross section, use the peaking approximation for σ_r :

$$\sigma_r(E_s', E_p') = F(-2s' \cdot p', T) \sigma_{Born} \equiv \sigma^{eff}(E_s', E_p') \quad \text{effective non-radiative cross section}$$

$$F(q^2, T) = 1 + 0.5772 bT$$

$$+ \frac{2\alpha}{\pi} \left[\frac{-14}{9} + \frac{13}{12} \ln \frac{-q^2}{m^2} \right]$$

← sum of the vacuum polarization and vertex correction

$$- \frac{\alpha}{2\pi} \ln^2 \left(\frac{E_s}{E_p} \right)$$

← a correction to the angle peaking approximation

$$+ \frac{\alpha}{\pi} \left[\frac{1}{6} \pi^2 - \phi \left(\cos^2 \frac{\theta}{2} \right) \right]$$

← Schwinger correction

- adding two external radiators to approximate the shape of internal bremsstrahlung, each of thickness t_r , one before and one after the scattering.;
- including ionization;

3

The radiative tail for continuum spectrum is:

$$\frac{d\sigma_{exp}}{d\Omega dE_p} = \left(\frac{R\Delta}{E_s} \right)^{T'} \left(\frac{\Delta}{E_p} \right)^{T'} \left(1 - \frac{\xi}{(1-2T')\Delta} \right) \sigma^{eff}(E_s, E_p) + \int_{E_{s \min}(E_p)}^{E_s - R\Delta} \sigma^{eff}(E_s', E_p) \left(\frac{E_s - E_s'}{E_p R} \right)^{T'} \left(\frac{E_s - E_s'}{E_s} \right)^{T'} \times \left[\frac{T'}{E_s - E_s'} \phi \left(\frac{E_s - E_s'}{E_s} \right) + \frac{\xi}{2(E_s - E_s')^2} \right] dE_s'$$

← $b(T/2 + t_r)$

$$+ \int_{E_p + \Delta}^{E_{p \max}} \sigma^{eff}(E_s, E_p') \left(\frac{E_p' - E_p}{E_p} \right)^{T'} \left(\frac{(E_p' - E_p)R}{E_s} \right)^{T'} \times \left[\frac{T'}{E_p' - E_p} \phi \left(\frac{E_p' - E_p}{E_p} \right) + \frac{\xi}{2(E_p' - E_p)^2} \right] dE_p'$$

Radiative cross section

folding formula:

$$\frac{d\sigma_{\text{exp}}}{d\Omega dE_p} = \left(\frac{R\Delta}{E_s}\right)^{T'} \left(\frac{\Delta}{E_p}\right)^{T'} \left(1 - \frac{\xi}{(1-2T')\Delta}\right) \sigma^{\text{eff}}(E_s, E_p) + \int_{E_{s \min}(E_p)}^{E_s - R\Delta} \sigma^{\text{eff}}(E'_s, E_p) \left(\frac{E_s - E'_s}{E_p R}\right)^{T'} \left(\frac{E_s - E'_s}{E_s}\right)^{T'} \times \left[\frac{T'}{E_s - E'_s} \phi\left(\frac{E_s - E'_s}{E_s}\right) + \frac{\xi}{2(E_s - E'_s)^2} \right] dE'_s$$

$$+ \int_{E_p + \Delta}^{E_{p \max}} \sigma^{\text{eff}}(E_s, E'_p) \left(\frac{E'_p - E_p}{E'_p}\right)^{T'} \left(\frac{(E'_p - E_p)R}{E_s}\right)^{T'} \times \left[\frac{T'}{E'_p - E_p} \phi\left(\frac{E'_p - E_p}{E'_p}\right) + \frac{\xi}{2(E'_p - E_p)^2} \right] dE'_p$$

- **Simulation**

- fit structure function from exist data;
- calculate Born cross section from structure function
- calculate the radiative cross section (then compare with measured cross section)

- **Unfolding data**

$$\sigma^{\text{eff}}(E_s, E_p) = \left(\frac{R\Delta}{E_s}\right)^{-T'} \left(\frac{\Delta}{E_p}\right)^{-T'} \left(1 - \frac{\xi}{(1-2T')\Delta}\right)^{-1}$$

$$\left[\frac{d\sigma_{\text{exp}}(E_s, E_p)}{d\Omega dE_p} - \int_{E_{s \min}(E_p)}^{E_s - R\Delta} (\dots) - \int_{E_p + \Delta}^{E_{p \max}(E_s)} (\dots) \right]$$

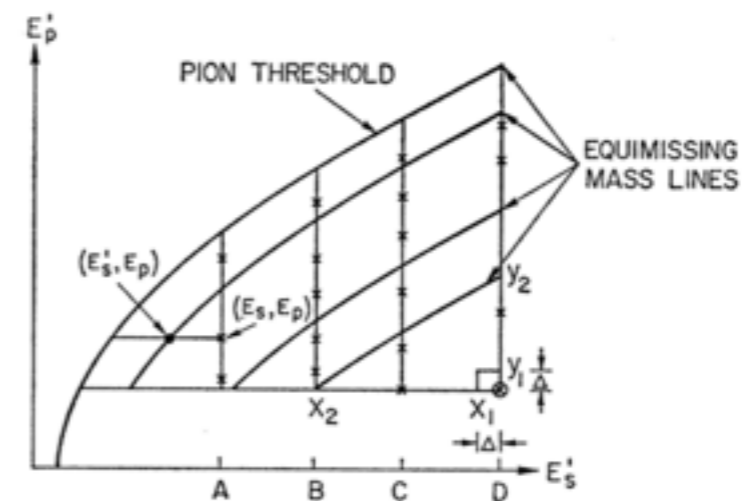


FIG. 4. Illustration of the unfolding procedure for each experimental data point. The cross represents a data point. We assume that there are four spectra corresponding to four incident energies A, B, C, and D available at one angle.

Input: E_s , E_p , θ

1. Correct E_p : E_p in radiative cross section formulas should be the measured E_p

subtract the energy loss due to the ionization and bremsstrahlung in materials between target and spectrometers; (windows before and after magnet)

2. Born cross section for DIS (deep inelastic scattering) and QE(quasi-elastic scattering):

$$\text{DIS: } F_1 F_2 \xrightarrow{Q^2 < 10 \text{ GeV}, W < 3 \text{ GeV}} F_1, F_2 \rightarrow \left(\frac{d\sigma}{d\Omega dE_p} \right)_B = \left(\frac{d\sigma}{d\Omega} \right)_{Mott} \left(W_2 + 2 \tan^2 \left(\frac{\theta}{2} \right) W_1 \right)$$

$$W_1 = \frac{F_1}{M_p}, W_2 = \frac{F_2}{\nu}$$

QE: y -scaling model;

3. Elastic tail: (Phys. Rev. D 12, 1884 A62) \longrightarrow tail

4. Quasi & DIS tail: (Phys. Rev. D 12, 1884 A82) \longrightarrow cs qt

5. Radiated cross section: $cs_Final = tail + cs_qt$;

Marathon radiative correction factor

x	W (GeV)	Q^2 [(GeV/c) ²]	E (GeV)	E' (GeV)	θ (deg)	π/e	Spectrometer
0.87	1.75	14.6	11.0	2.07	47.1	11	BBS
0.83	1.98	14.8	11.0	1.48	57.1	92	BBS
0.79	2.16	14.2	11.0	1.41	57.1	121	BBS
0.75	2.30	13.3	11.0	1.58	51.9	66	BBS
0.71	2.45	12.7	11.0	1.50	51.9	89	BBS
0.67	2.58	11.7	11.0	1.67	47.1	52	BBS
0.63	2.68	10.8	11.0	1.90	42.0	27	BBS
0.59	2.82	10.2	11.0	1.80	42.0	39	BBS
0.55	2.60	7.22	11.0	4.00	23.4	1	HRS
0.51	2.71	6.70	11.0	4.00	22.5	1	HRS
0.47	2.80	6.17	11.0	4.00	21.6	1	HRS
0.43	2.89	5.65	11.0	4.00	20.6	1	HRS
0.39	2.98	5.12	11.0	4.00	19.6	1	HRS
0.35	3.07	4.60	11.0	4.00	18.6	2	HRS
0.31	3.15	4.07	11.0	4.00	17.5	2	HRS
0.27	3.24	3.55	11.0	4.00	16.3	3	HRS
0.23	3.32	3.02	11.0	4.00	15.1	3	HRS

- For each kinematics: 9
- E_p : 3.75 - 4.24,
step: 0.01 50
 - θ : ± 3 deg;
step: 0.5 deg; 12
 - $\theta < 23.1$ or $E_p < 3.9$
(very roughly by experience)

total: 4800

Marathon radiative correction factor

why cut on E_p & theta:

when calculate inelastic tail, it needs to do integrands over E_p and E_s :

$$\begin{aligned} \sigma_{\text{et}} \equiv \left(\frac{d^2\sigma}{d\Omega dE_p} \right)_q &= \left(\frac{R\Delta E}{E_s} \right)^{b(t_b+t_r)} \left(\frac{\Delta E}{E_p} \right)^{b(t_a+t_r)} \left[1 - \frac{\xi/\Delta E}{1 - b(t_a+t_b+2t_r)} \right] \bar{\sigma}_q(E_s, E_p) \\ &+ \int_{E_s - R\Delta E}^{E_s - \min(E_p)} \bar{\sigma}_q(E'_s, E_p) \left(\frac{E_s - E'_s}{E_p R} \right)^{b(t_a+t_r)} \left(\frac{E_s - E'_s}{E_s} \right)^{b(t_b+t_r)} \left[\frac{b(t_b+t_r)}{E_s - E'_s} \phi \left(\frac{E_s - E'_s}{E_s} \right) + \frac{\xi}{2(E_s - E'_s)^2} \right] dE'_s \\ &+ \int_{E_p + \Delta E}^{E_p \text{ max}} \bar{\sigma}_q(E_s, E'_p) \left(\frac{E'_p - E_p}{E'_p} \right)^{b(t_a+t_r)} \left[\frac{(E'_p - E_p)R}{E_s} \right]^{b(t_b+t_r)} \left[\frac{b(t_a+t_r)}{E'_p - E_p} \phi \left(\frac{E'_p - E_p}{E'_p} \right) + \frac{\xi}{2(E'_p - E_p)^2} \right] dE'_p, \end{aligned} \quad (\text{A82})$$

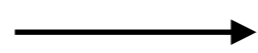
$$\begin{aligned} E_{p \text{ max}}(E'_s) \\ = (2ME'_s - 2Mm_\pi - m_\pi^2) / [2M + 2E'_s(1 - \cos \theta)]. \end{aligned}$$

if $E_s = 11 \text{ GeV}$, $\theta = 23.9$,

then: $E_p(\text{max}) = 5.41$

$Q^2 = 10.21$;

$W^2 = 1.51$



out of the range that F1F209 can calculate

Marathon radiative correction factor

other problems:

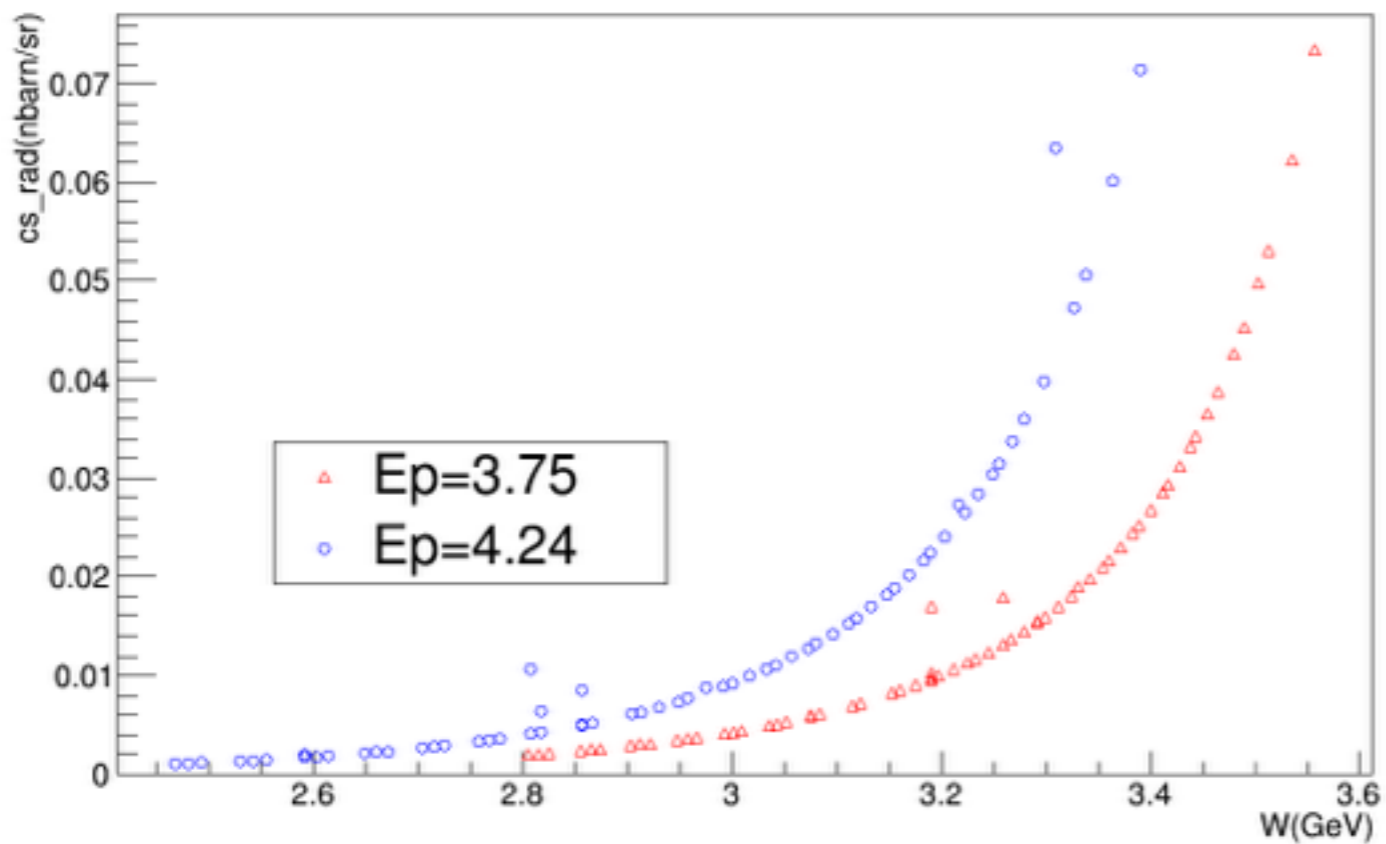
To calculate quasi-elastic tail, need QE cross section.

In XEMC, cs_{qe} is computed from a y -scaling model, given by a fit to the scaling function $F(y)$:

$$F(y) = \frac{d\sigma}{d\Omega dv} \frac{1}{Z\sigma_p + N\sigma_N} \frac{q}{\sqrt{M^2 + (y+q)^2}}$$

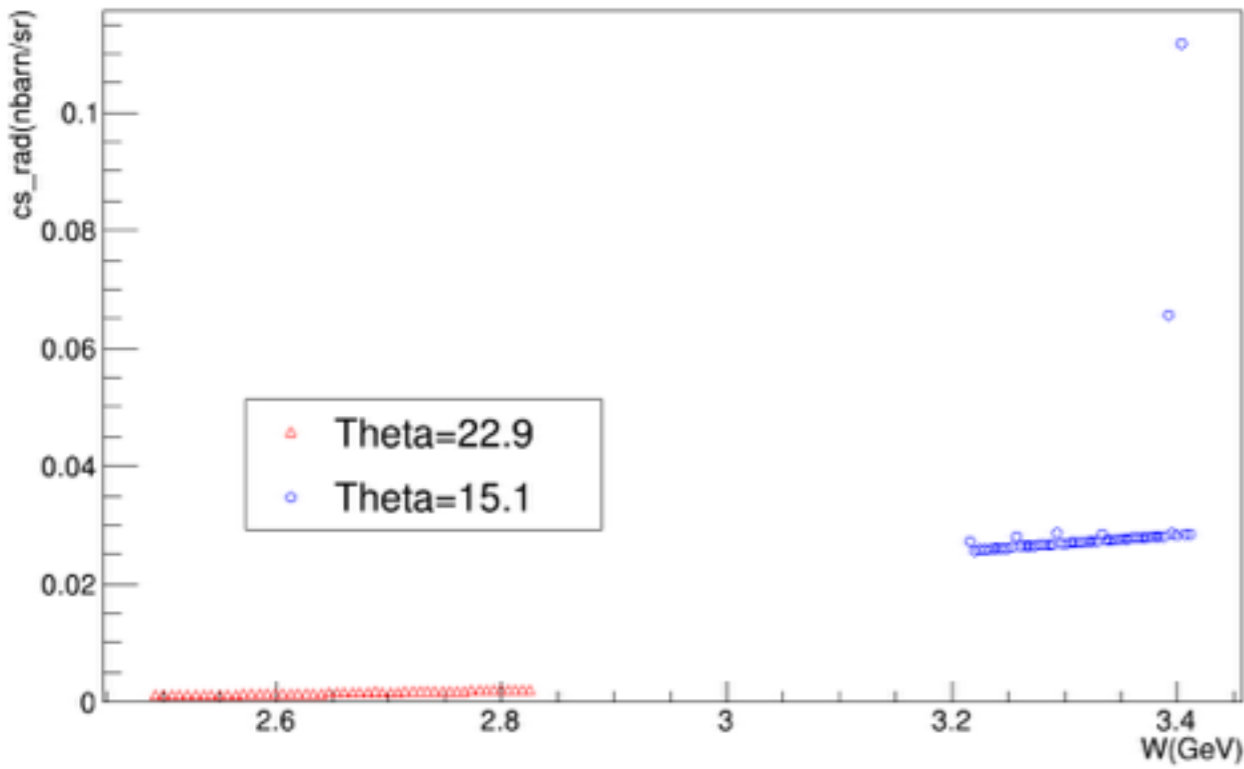
where $F(y)$ is given by world fit. But so far I didn't have the parameters to get $F(y)$ for tritium

CS vs W

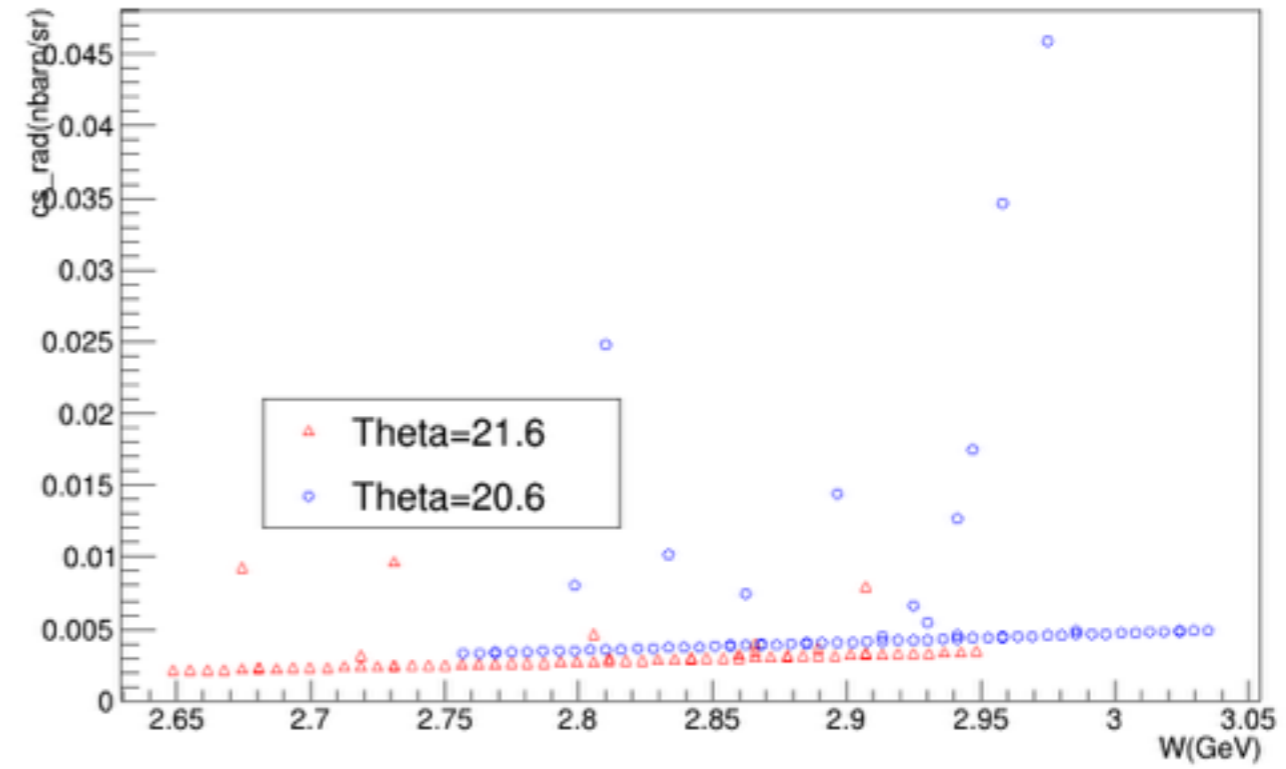


Tritium target

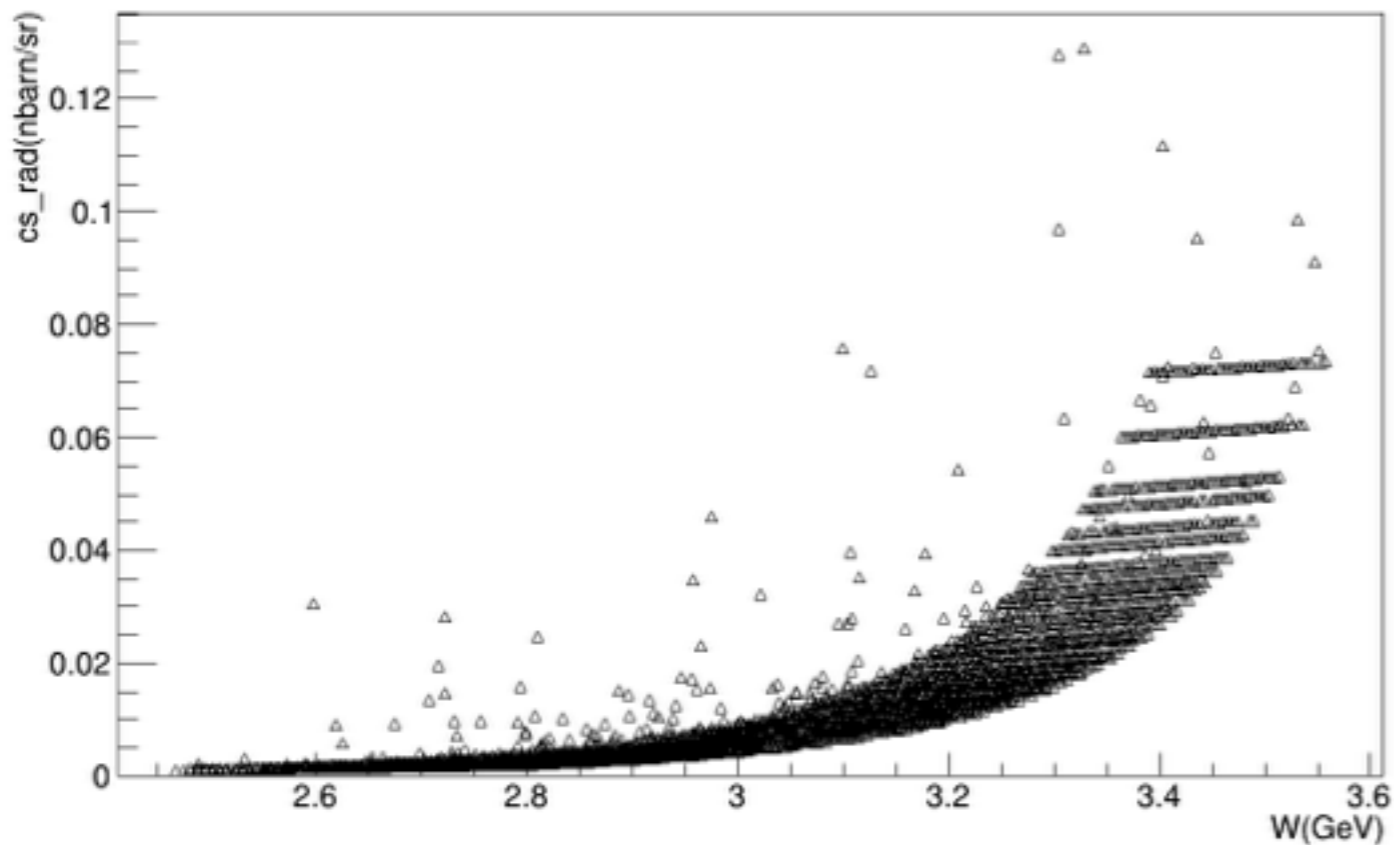
CS vs W



CS vs W



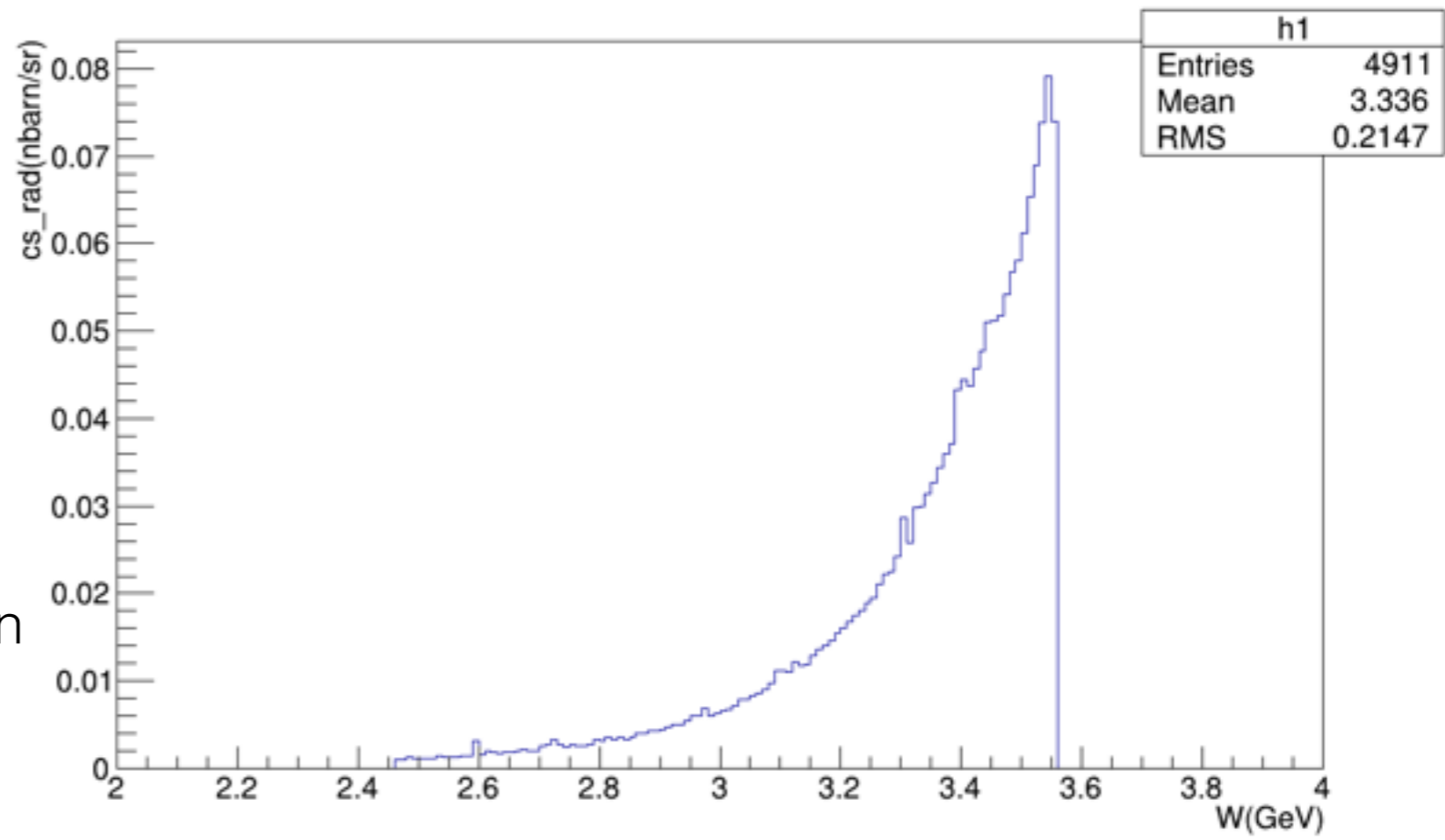
cs vs W



← TGraph

Tritium target

cs vs W



TH1*

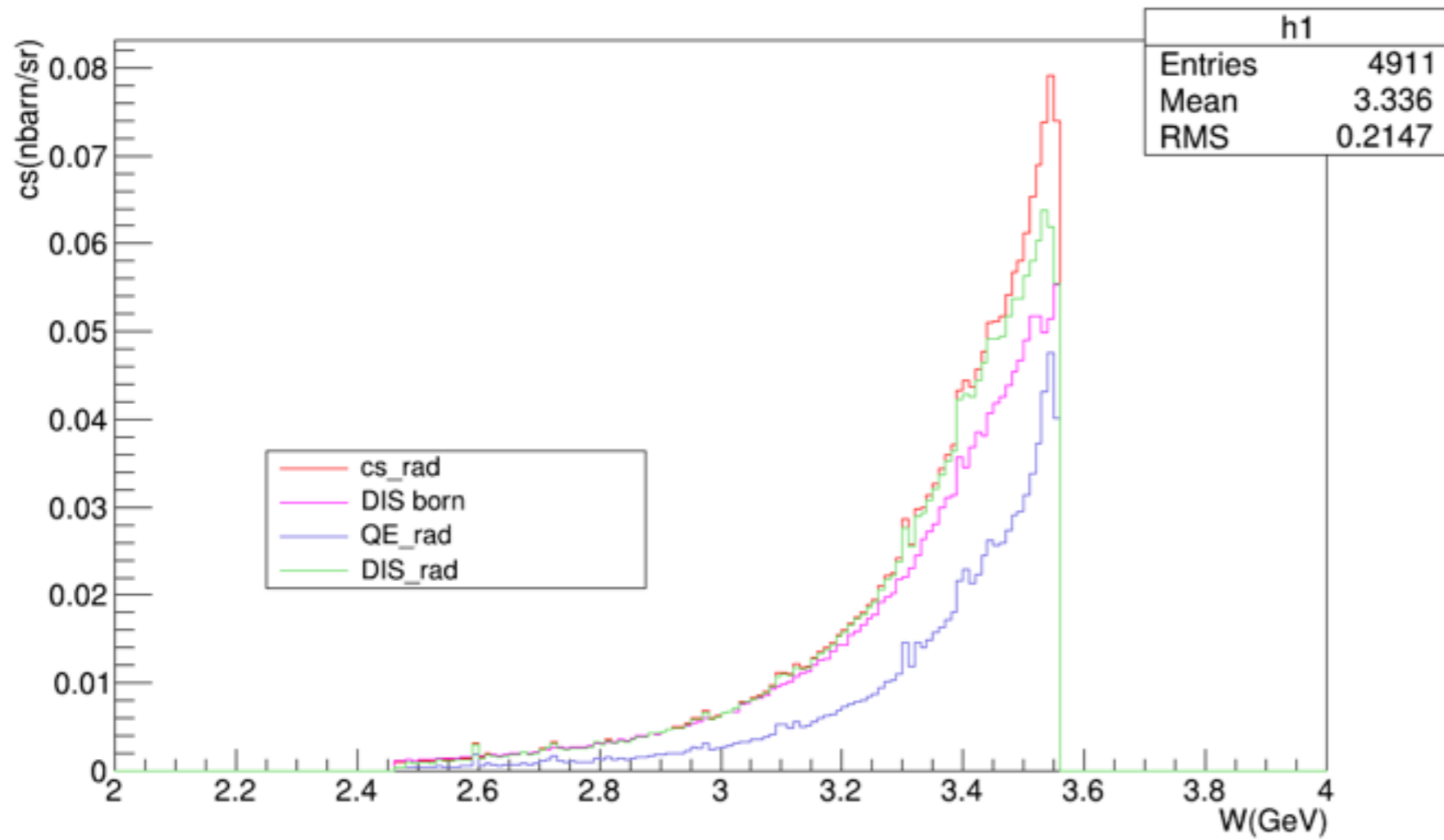


weighted by average cs



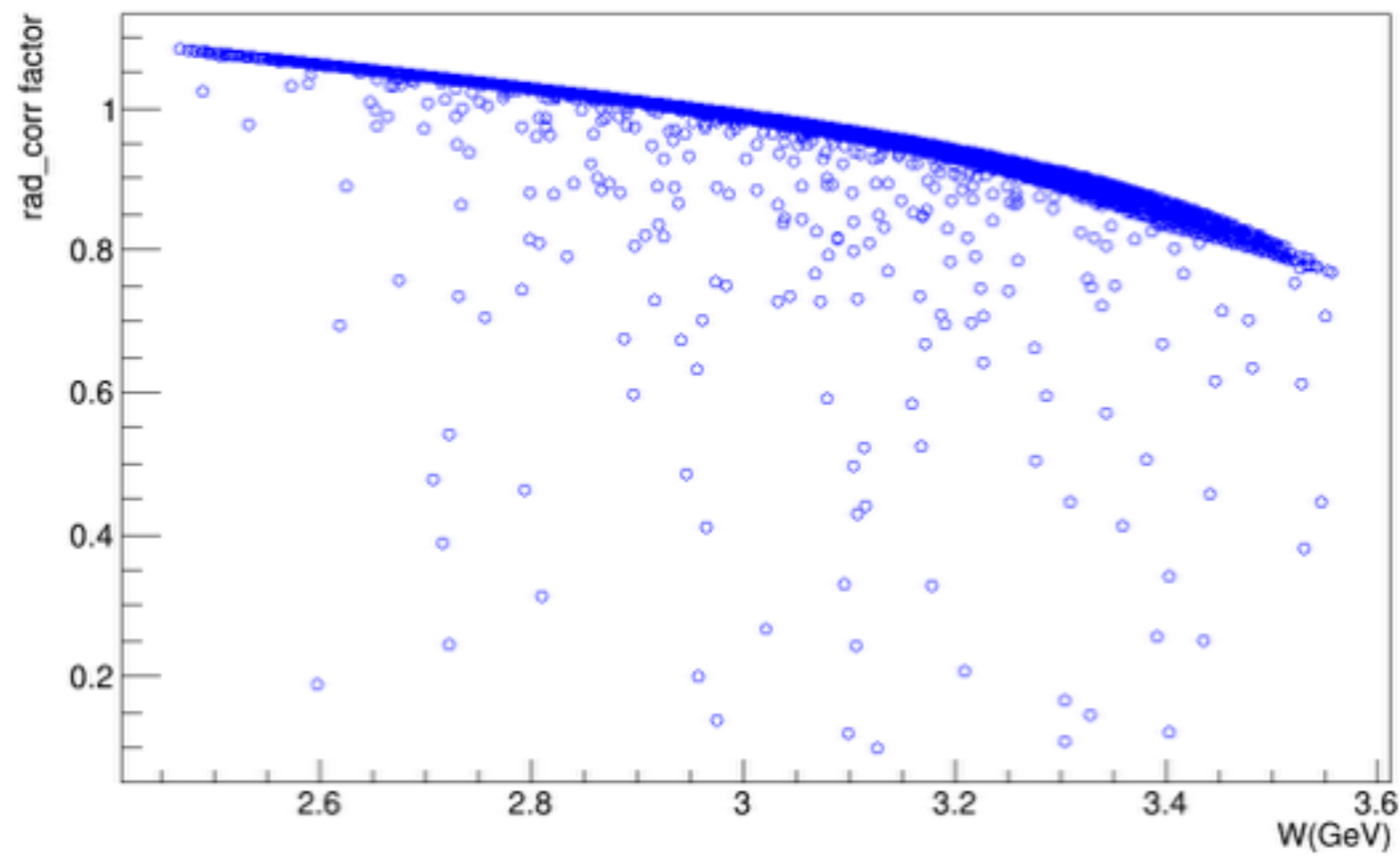
not accurate, should do interpolation

cs vs W



Tritium target

correction factor vs W



Reference

- [1]. L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. 41, 205 (1969); [\(idea, internal\)](#)
- [2]. Phys. Rev. D 12, 1884 (1975) ; [\(clear formulas\)](#)
- [3]. T. S. Tsai, SLAC-PUB-848 (1971); [\(detailed\)](#)
- [4]. T. S. Tsai, Phys. Rev. 122, 1898 (1961); [\(elastic peak\)](#)