Hypernuclear structure and production with antisymmetrized molecular dynamics

Masahiro ISAKA, Hosei University

Grand challenges of hypernuclear physics

Interaction: "baryon-baryon interaction"

- **2 body interaction between baryons (Y: hyperon, N: nucleon)**
  - hyperon-nucleon (YN)
  - hyperon-hyperon (YY)

} Major issues in hypernuclear physics

Structure: "many-body system of nucleons and hyperon"

- **Addition of hyperon as an impurity in (hyper)nuclei**
  - No Pauli exclusion between N and Y
  - YN interaction is different from NN

→ Structure changes
Unique structure, ··· etc.

Today: "deformation of hypernuclei"

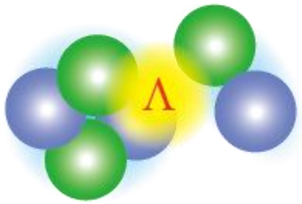
Structure of Λ hypernuclei

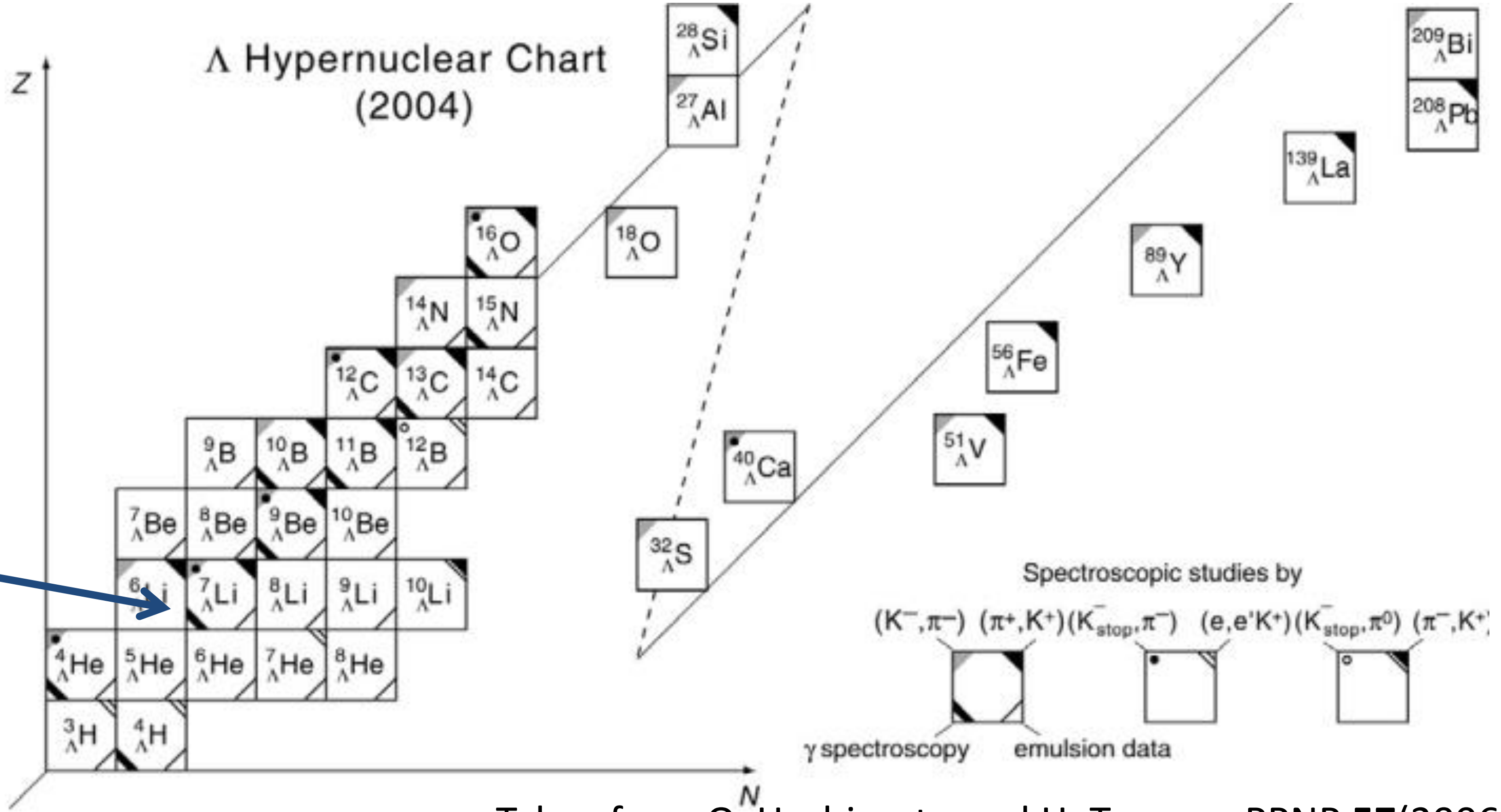
◆Λ hypernuclei observed so far

- **Concentrated in light Λ hypernuclei**
- **Most have well-developed cluster structure**

Light Λ hypernuclei

Developed cluster





Taken from O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

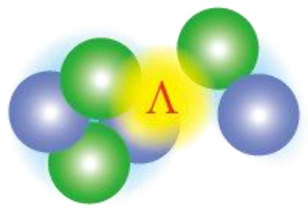
Structure of Λ hypernuclei

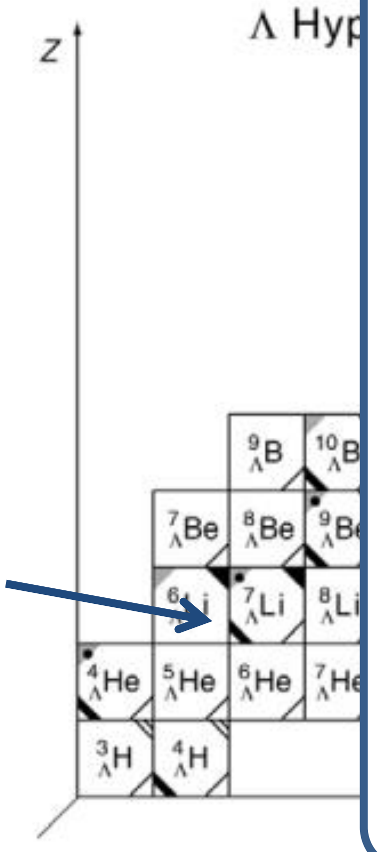
◆Λ hypernuclei observed so far

- Concentrated in light Λ hypernuclei
- Most have well-developed cluster structure

Light Λ hypernuclei

Developed cluster





Example of "impurity effect"

Example: $^{7}_{\Lambda}$Li

Motoba, *et al.*, PTP**70**,189 (1983)
Hiyama, *et al.*, PRC**59** (1999), 2351.
Tanida, *et al.*, PRL**86** (2001), 1982.

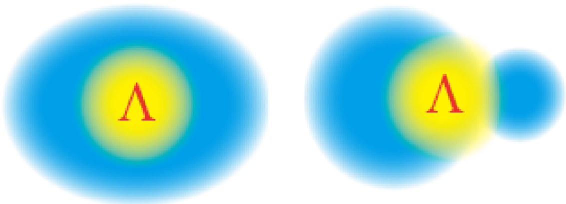
- Λ reduces inter-cluster distance between α + *d* of the core nucleus ^{6}Li
- Confirmed through B(E2) reduction

Toward heavier and exotic Λ hypernuclei

◆Experiments at J-PARC, JLab, *etc*.

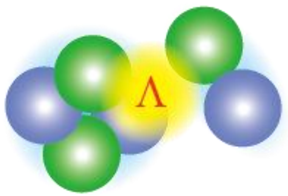
- **Heavier(*sd*-shell) & n-rich hypernuclei can be produced**
- **Various structures will appear**

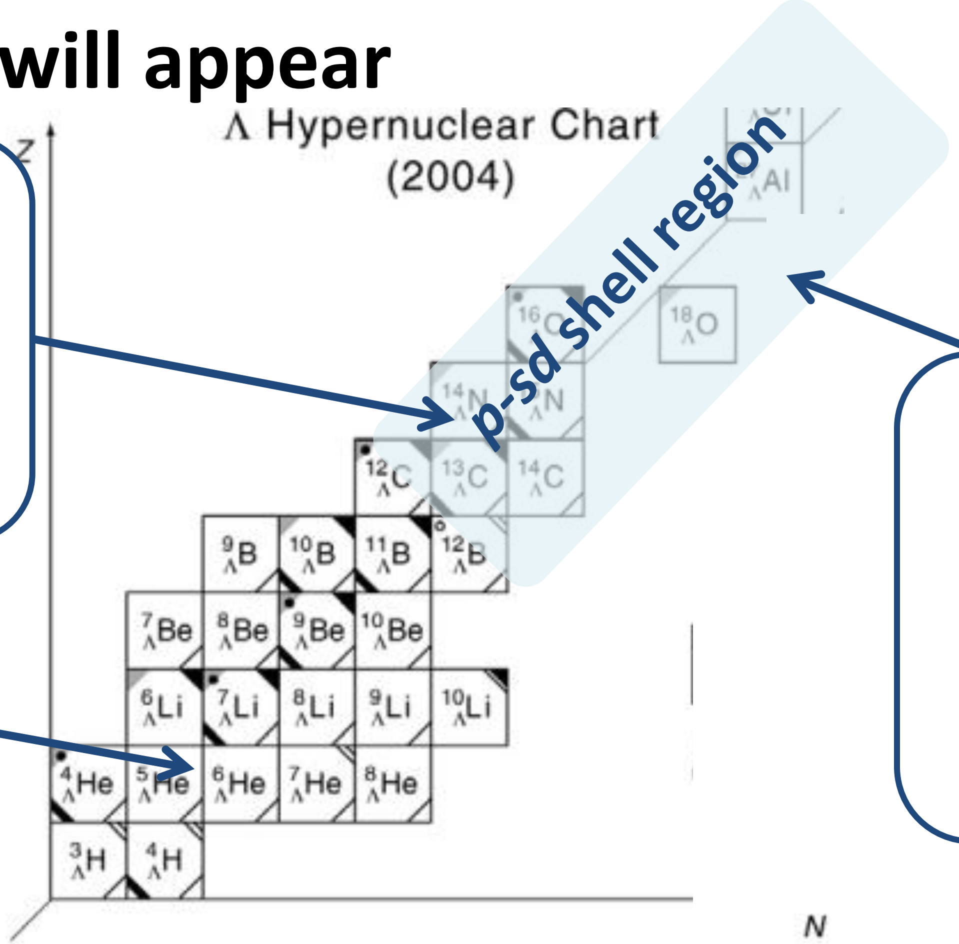
Coexistence of structures



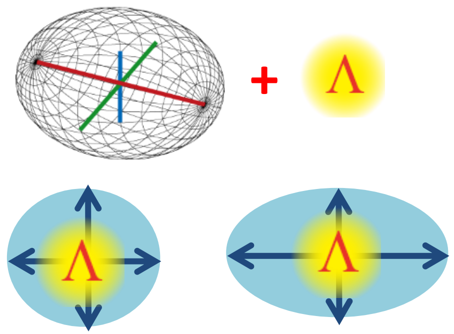
Light Λ hypernuclei

Developed cluster





Various deformations



Today: "deformation of hypernuclei"

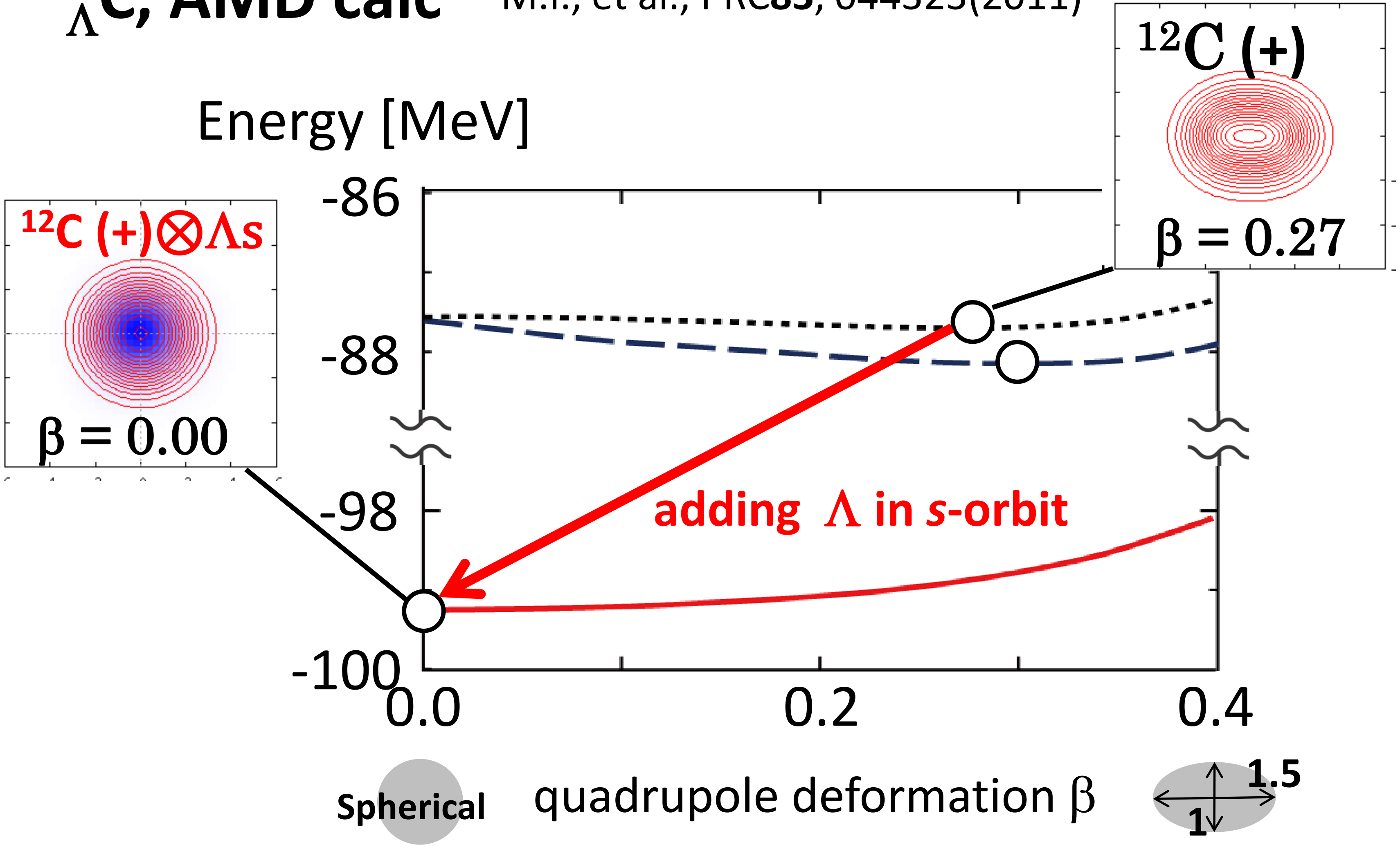
What is expected in deformed Λ hypernuclei

- **Deformation change**
  - Λ particle can change nuclear deformation
- **Difference of B_Λ depending on nuclear deformation**
  - Energy shifts in excitation spectra
- **Coupling of Λ to deformed nuclei shows unique structure**
  - For example, rotational band, mixing of configuration, … etc.

Deformation change by Λ particle

◆Λ particle in s orbit reduces nuclear deformation

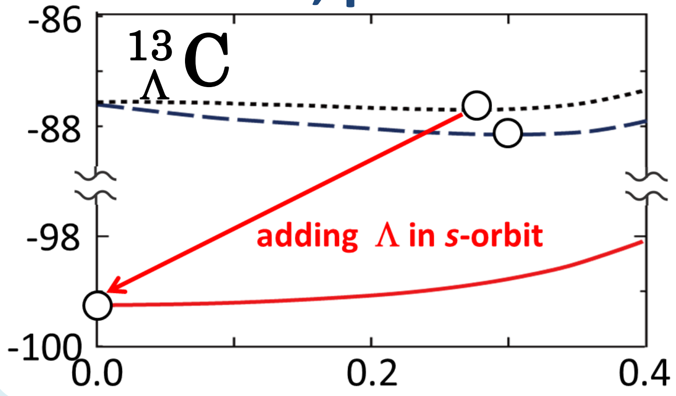
Example: $^{13}{}_{\Lambda}C$, AMD calc M.I., et al., PRC**83**, 044323(2011)



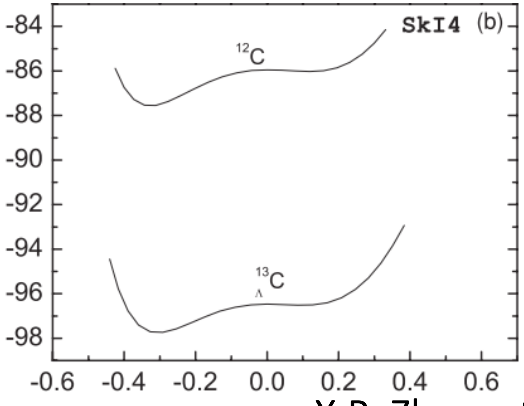
Deformation change by Λ particle

Many authors predict the deformation change by Λ in *s*-orbit

AMD, present

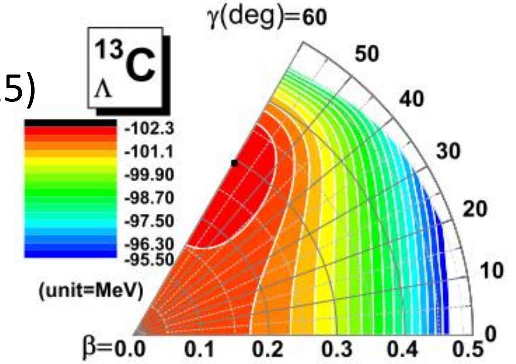


Skyrme-Hartree-Fock (SHF)



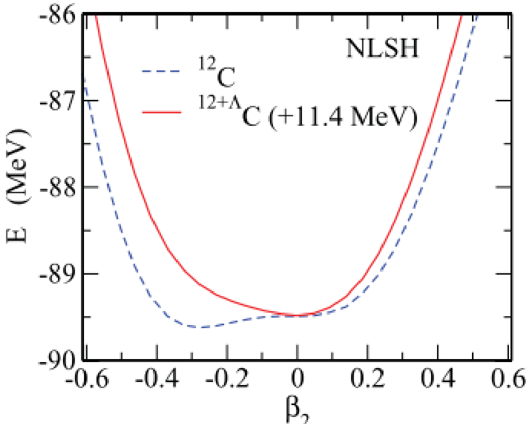
X.R. Zhou, et al., PRC**76**, 034312('07)

J.W. Cui, et al, PRC**91**,054306('15)

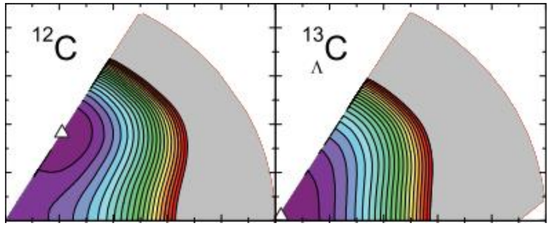


Relativistic mean-field (RMF)

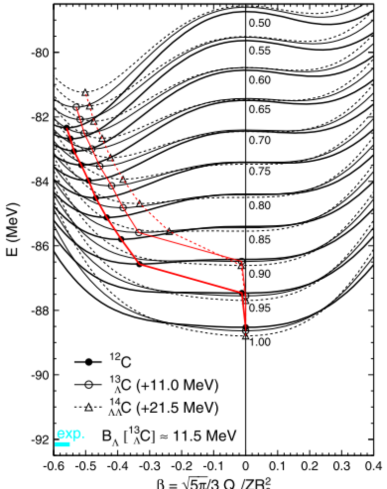
Win and Hagino, PR C**78**, 054311('08)



B.N. Lu, et al., PRC**84**, 014328 ('11)



RMF & SHF



H. J. Schulze, et al., PTP**123**, 569('10)

Deformations/level structure with beyond-mean-field

J.W. Cui, X.R. Zhou, H.J. Schulze, PRC**91**,054306('15)

H. Mei, K. Hagino, J.M. Yao, T. Motoba, PRC91, 064305('15)

etc.

What is expected in deformed Λ hypernuclei

- **Deformation change**
  - Λ particle can change nuclear deformation
- **Difference of B_Λ depending on nuclear deformation**
  - Energy shifts in excitation spectra
- **Coupling of Λ to deformed nuclei shows unique structure**
  - For example, rotational band, mixing of configuration, … etc.

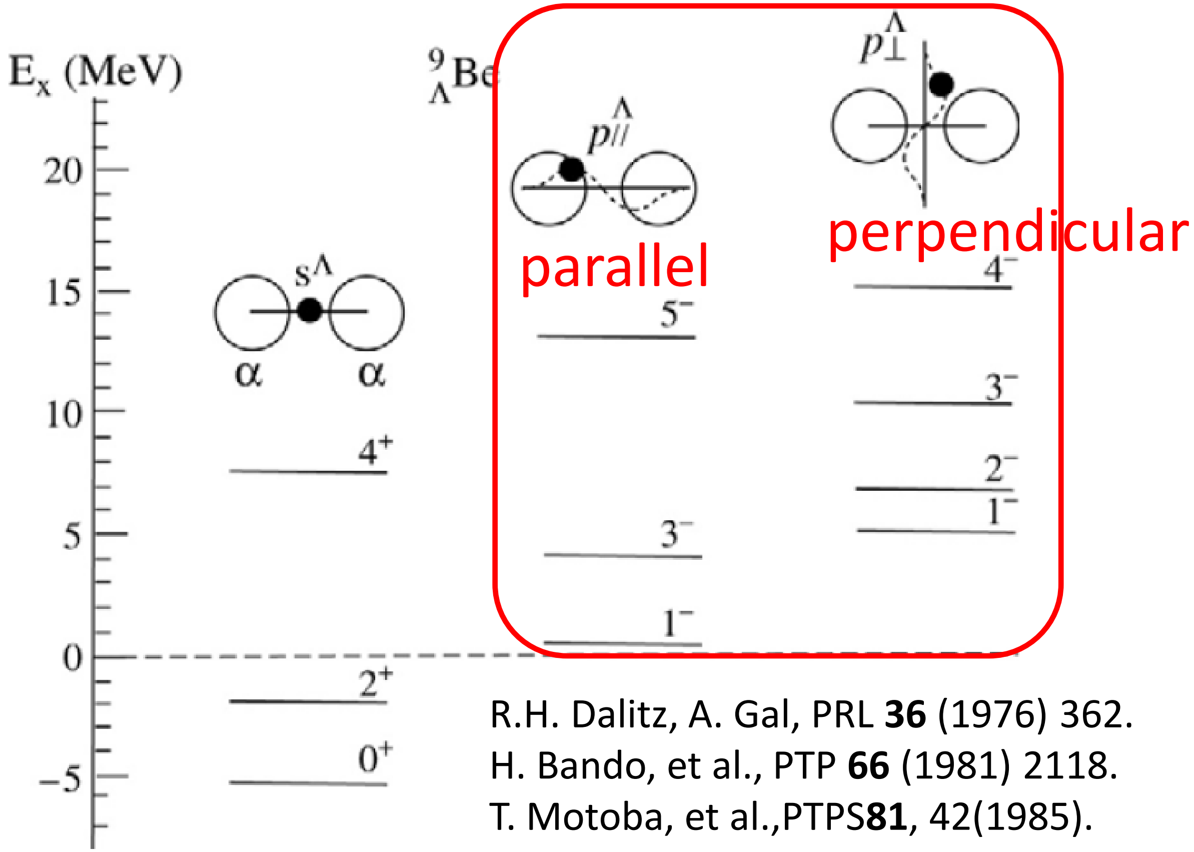
Today
- Triaxial deformation of Mg nuclei: $^{27}_{\Lambda}$Mg, future exp at JLab
- Ongoing project: production cross section with AMD wf

Coupling of Λ in *p*-orbit: *p*-states of $^9{}_\Lambda$Be

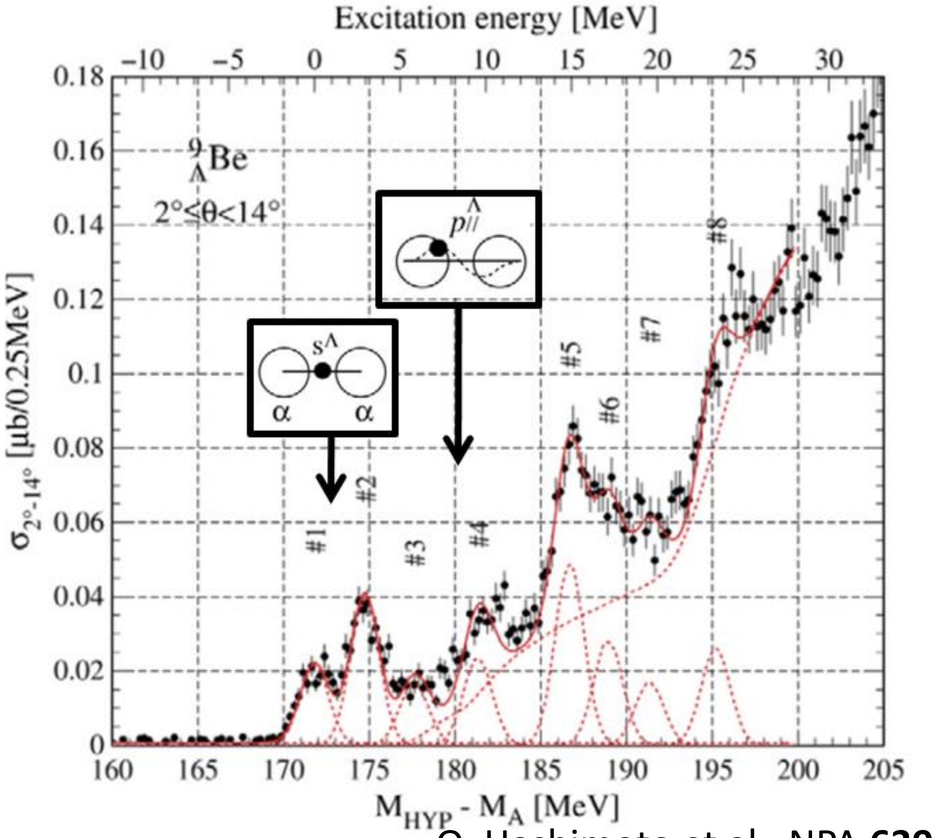
$^9{}_\Lambda$Be: **axially symmetric** 2α clustering

Two rotational bands as *p*-states
- Anisotropic *p* orbit of Λ hyperon
- Axial symmetry of 2α clustering

→ *p*-orbit parallel to/perpendicular to the 2α clustering



R.H. Dalitz, A. Gal, PRL **36** (1976) 362.
H. Bando, et al., PTP **66** (1981) 2118.
T. Motoba, et al.,PTPS**81**, 42(1985).



O. Hashimoto et al., NPA **639** (1998) 93c.

Theoretical Framework: HyperAMD

M.I., *et al.*, PRC**83**(2011) 044323
M. I., *et al.*, PRC**83**(2011) 054304

HyperAMD: Antisymmetrized Molecular Dynamics for hypernuclei

◆Hamiltonian

$$\hat{H} = \hat{T}_N + \hat{V}_{NN} + \hat{T}_\Lambda + \hat{V}_{\Lambda N} - \hat{T}_g$$

NN : Gogny D1S
ΛN : YNG interaction (ESC14, NSC97f)

◆Wave function

- **Nucleon part : Slater determinant**
  Spatial part of s.-p. w.f. is described as Gaussian packets

$$\varphi_N(\vec{r}) = \frac{1}{\sqrt{A!}} \det\left[\varphi_i(\vec{r}_j)\right]$$

$$\varphi_i(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \nu_\sigma (r - Z_i)_\sigma^2\right] \chi_i \eta_i$$

$$\chi_i = \alpha_i \chi_\uparrow + \beta_i \chi_\downarrow$$

- **Single-particle w.f. of Λ hyperon:**
  Superposition of Gaussian packets

$$\varphi_\Lambda(r) = \sum_m c_m \varphi_m(r)$$

$$\varphi_m(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \mu \nu_\sigma (r - z_m)_\sigma^2\right] \chi_m$$

$$\chi_m = a_m \chi_\uparrow + b_m \chi_\downarrow$$

- **Total w.f. :**

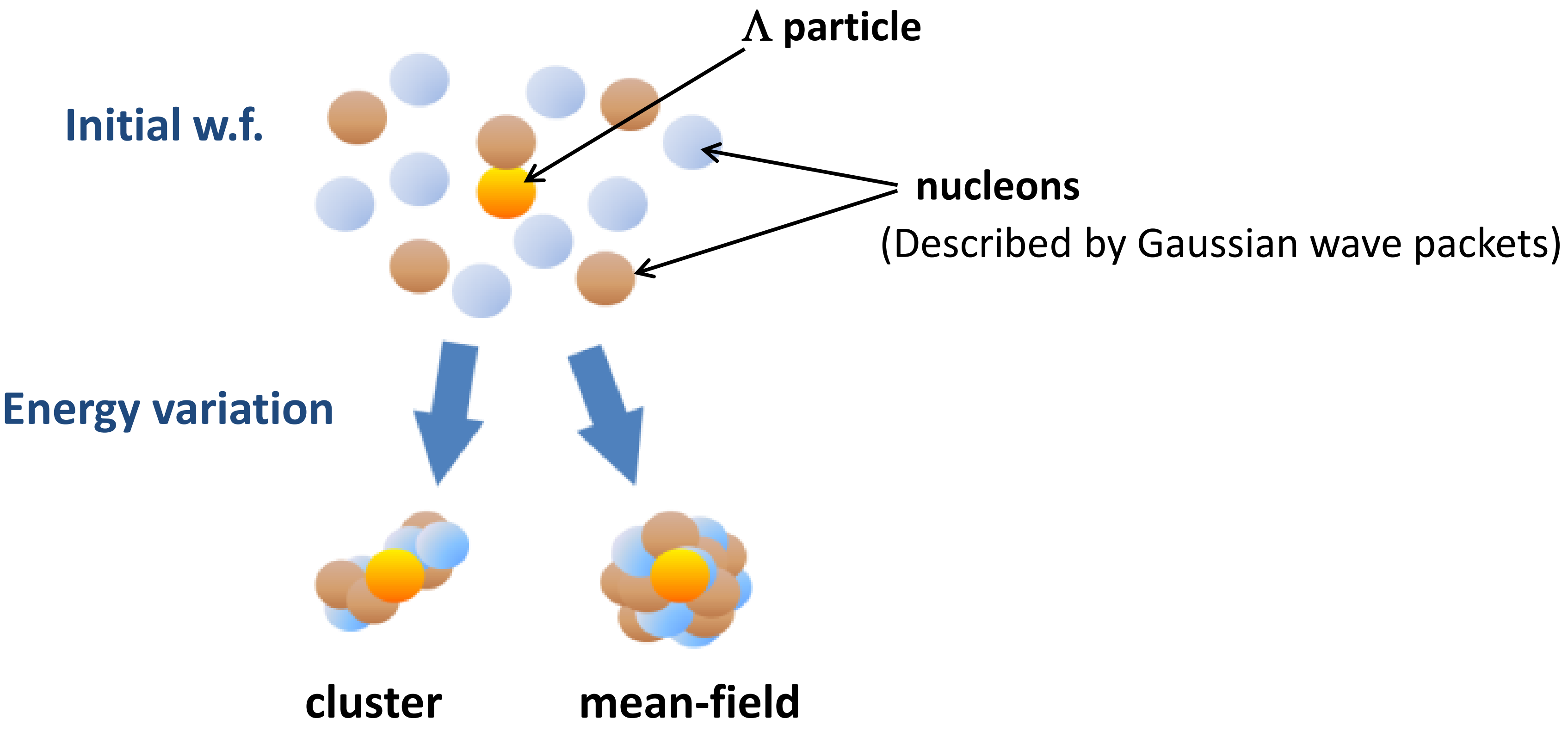
$$\psi(\vec{r}) = \sum_m c_m \varphi_m(r_\Lambda) \otimes \frac{1}{\sqrt{A!}} \det\left[\varphi_i(\vec{r}_j)\right]$$

Theoretical framework: HyperAMD

◆Procedure of the calculation

Variation

- Imaginary time development method: $\frac{dX_i}{dt} = \frac{\kappa}{\hbar}\frac{\partial H^{\pm}}{\partial X_i^{*}}$ $\quad \kappa < 0$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$



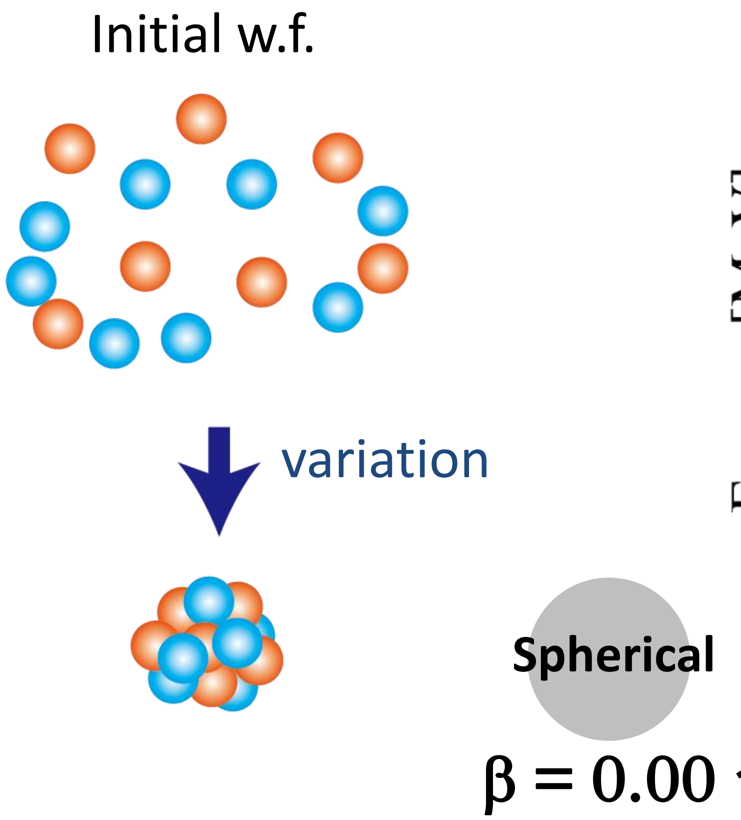
Theoretical framework: HyperAMD

◆Procedure of the calculation

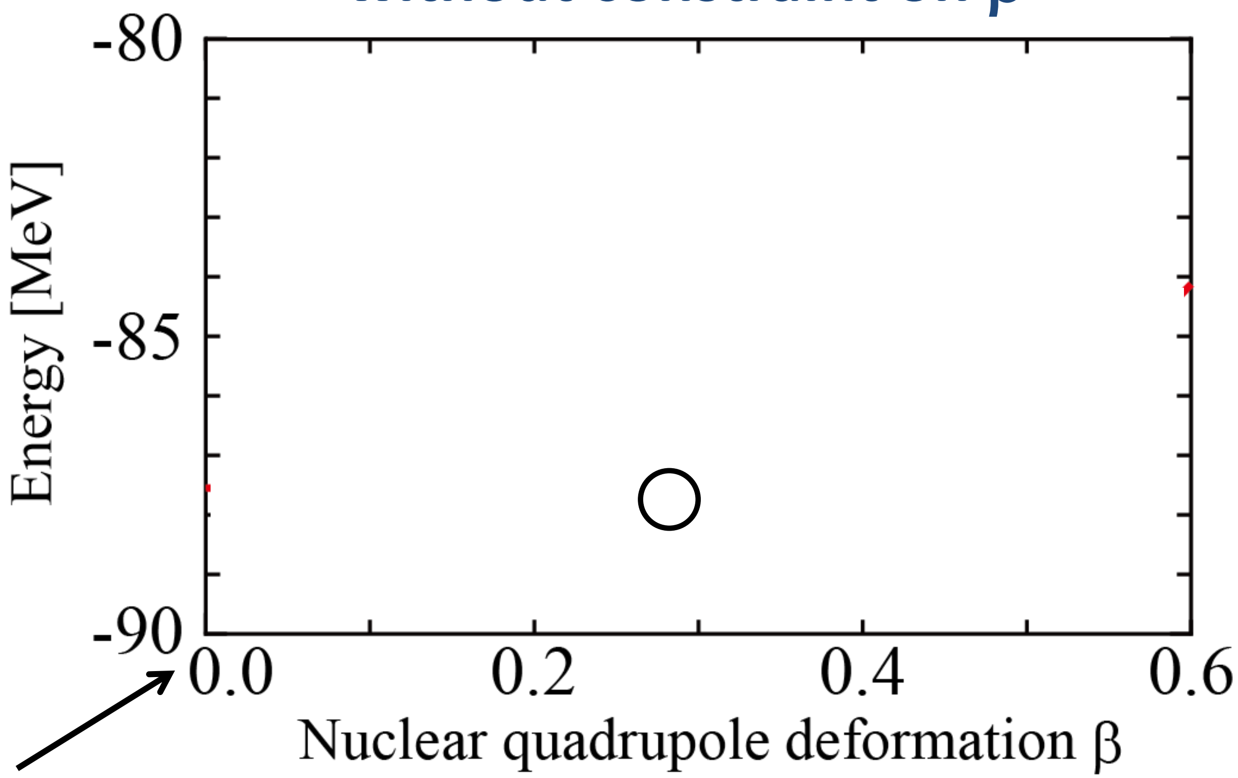
- **Energy variation with constraint on nuclear quadrupole deformation**

Described by (β, γ)

e.g.) ^{12}C



without constraint on β



Theoretical framework: HyperAMD

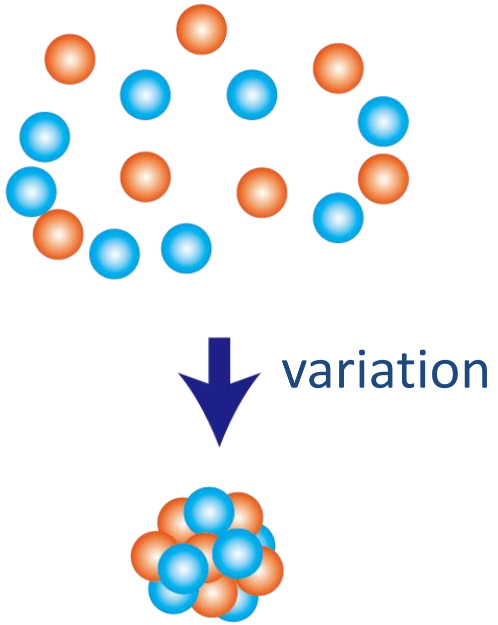
◆Procedure of the calculation

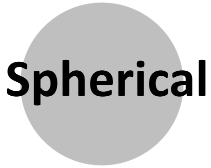
- **Energy variation with constraint on nuclear quadrupole deformation**

Described by (β, γ)

e.g.) ^{12}C

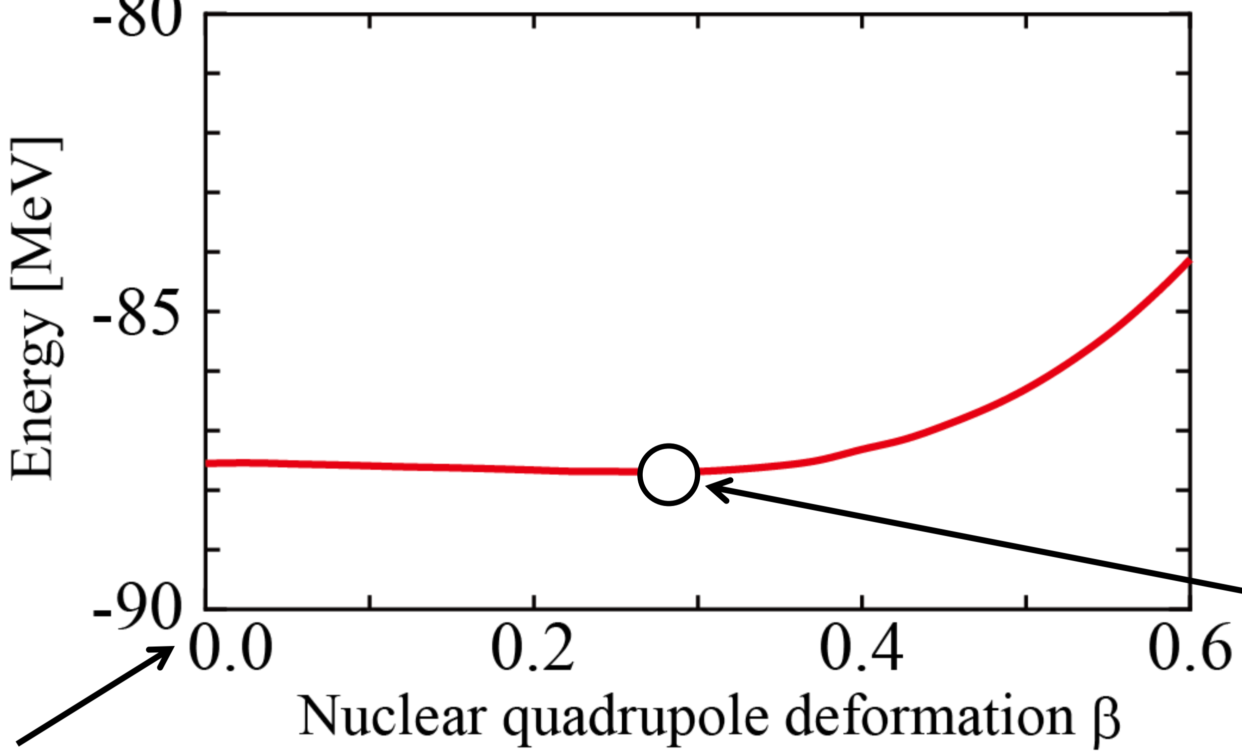
Initial w.f.

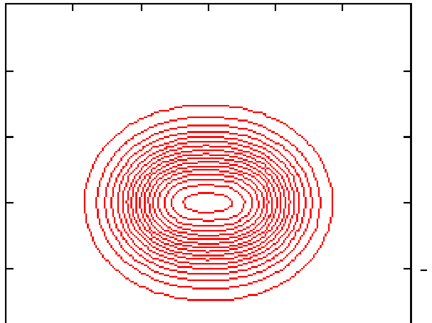




$\beta = 0.00$

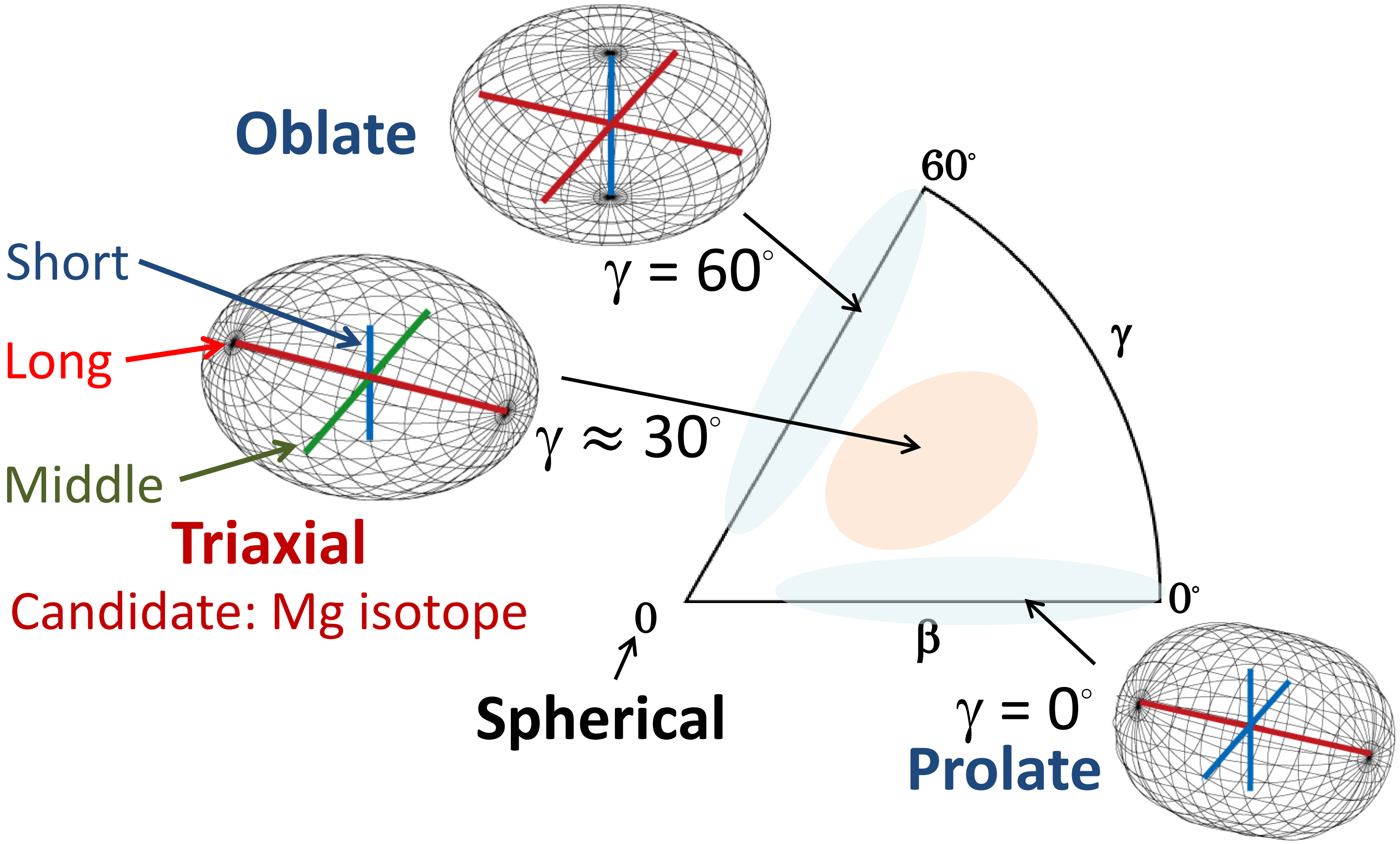
with constraint on β

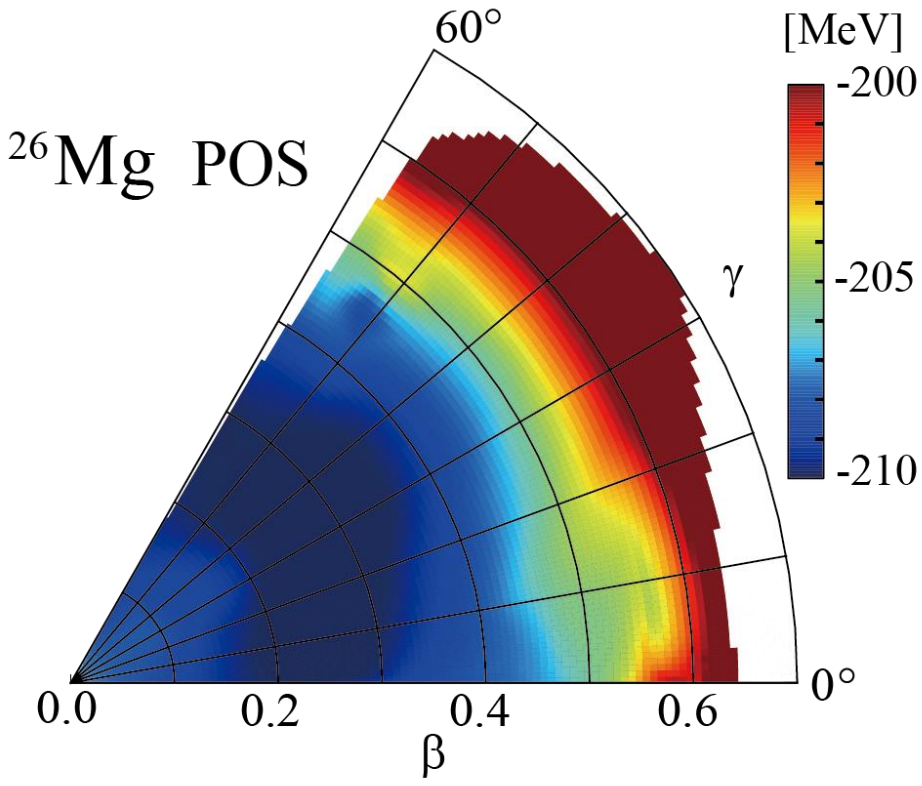




$\beta = 0.27$

Quadrupole deformation (β, γ)





- Energy variation is performed at each (β, γ)
- *p* states are obtained by constraint on Λ single particle wf: $V_f = \lambda \sum_f |\varphi_f\rangle\langle\varphi_f|$

Theoretical Framework: HyperAMD

M.I., *et al.*, PRC**83**(2011) 044323
M. I., *et al.*, PRC**83**(2011) 054304

◆Procedure of the numerical calculation

Variation

- Imaginary time development method: $\frac{dX_i}{dt} = \frac{\kappa}{\hbar}\frac{\partial H^{\pm}}{\partial X_i^*} \qquad \kappa < 0$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$

↓

Angular Momentum Projection

$$\left|\Phi_K^s; JM\right\rangle = \int d\Omega D_{MK}^{J*}(\Omega) R(\Omega) \left|\Phi^{s+}\right\rangle$$

↓

Generator Coordinate Method (GCM)

- Superposition of intrinsic wave functions with different configuration
- Diagonalization of $H_{sK,s'K'}^{J\pm}$ and $N_{sK,s'K'}^{J\pm}$

$$H_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s; J^{\pm}M \right| \hat{H} \left| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$

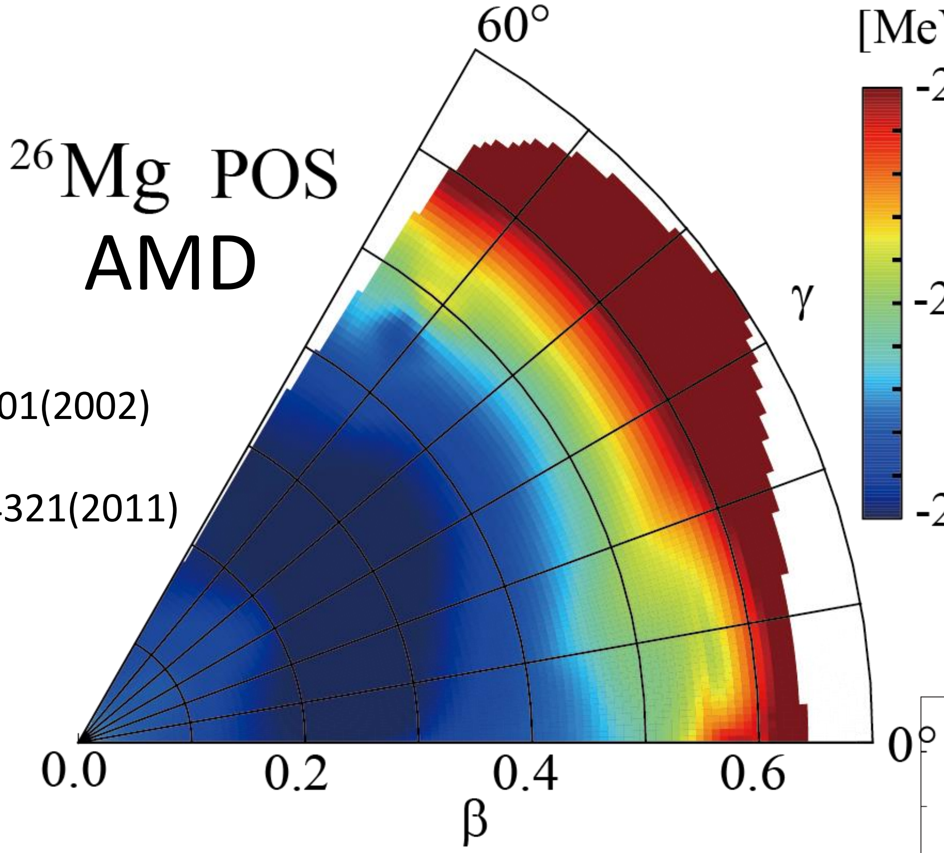
$$N_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s; J^{\pm}M \middle| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$

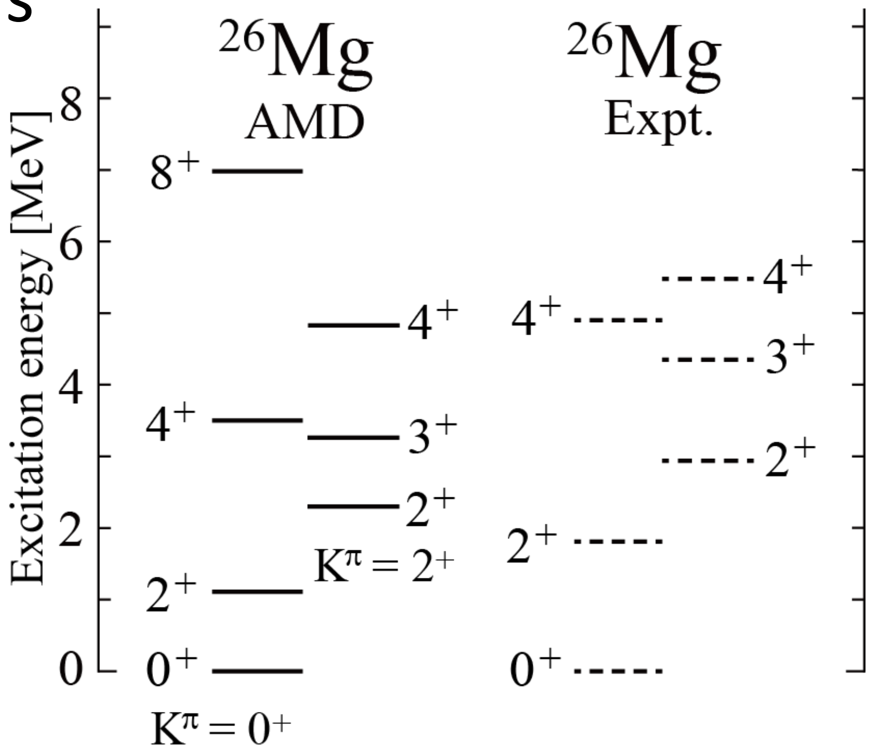
$$\left|\Psi^{J\pm M}\right\rangle = \sum_{sK} g_{sK} \left|\Phi_K^s; J^{\pm}M\right\rangle$$

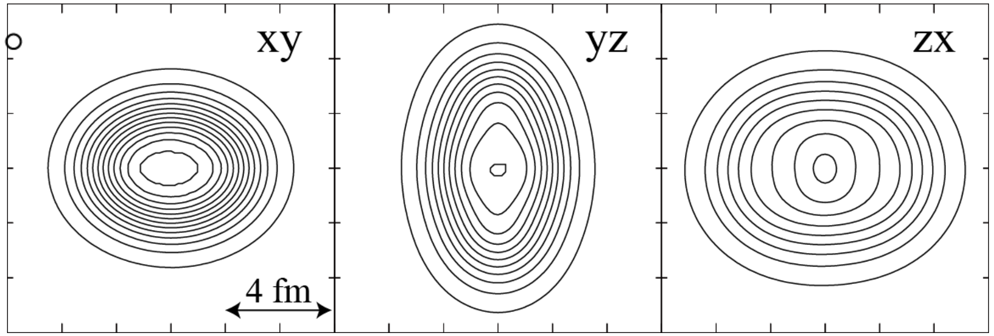
Deformation of Mg nuclei

^{26}Mg

- Shell gap in Nilsson diagram: Z=12 (prolate) vs. N=14 (oblate) → triaxial
- β,γ-soft nature is discussed by several authors





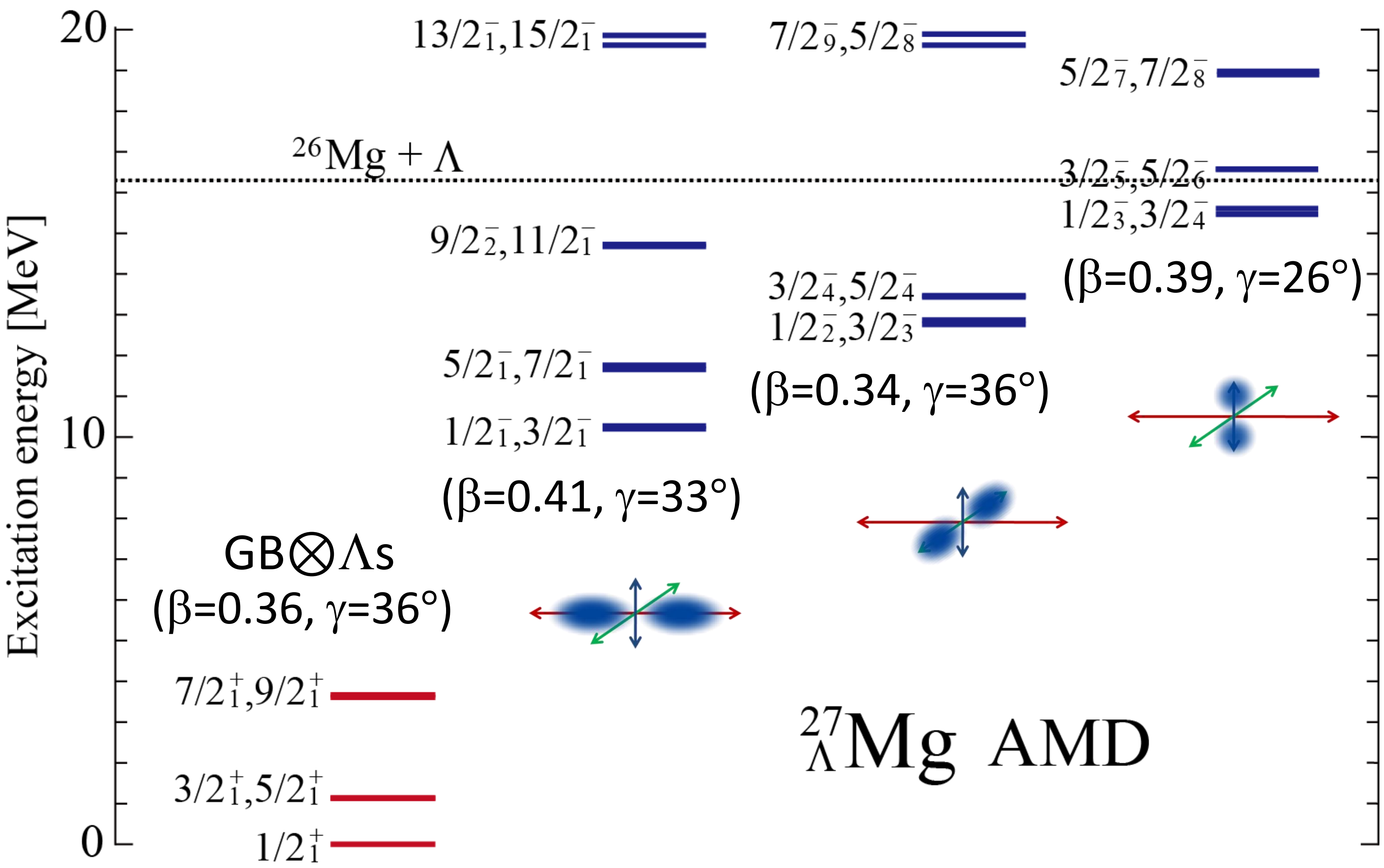


Terasaki et al., NPA**621**, 706(1997)
Rodrıguez-Guzman et al., NPA**709**, 201(2002)
Peru et al., PRC**77**, 044313(2008)
Hinohara, Kanada-En'yo, PRC**83**, 014321(2011)

After angular momentum projection,
energy minimum appear with triaxial deformation

Results: $^{27}{}_{\Lambda}$Mg

- **3 bands are obtained by Λ in *p*-orbit** → **Splitting of the *p* states**

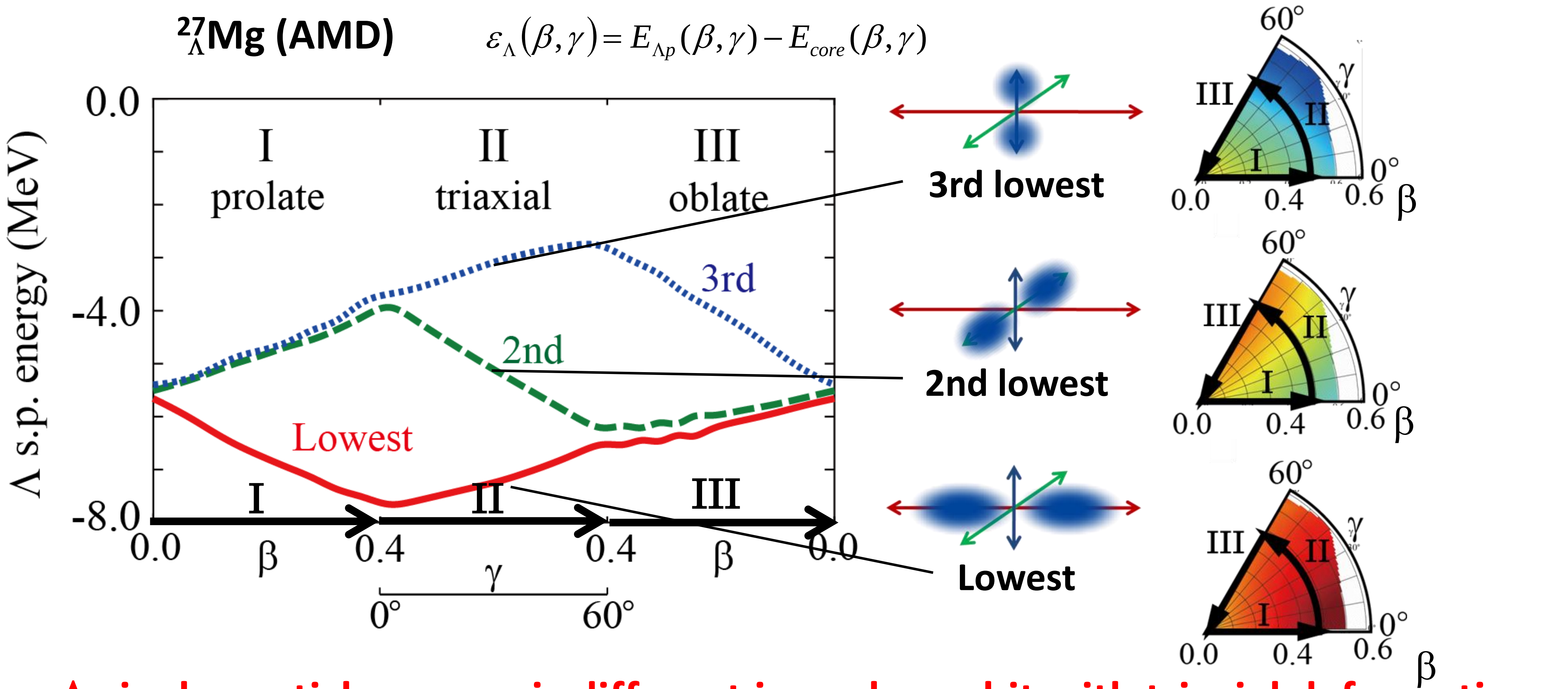


cf. ^{26}Mg ground state:
β=0.41, γ=33°

Results : Single particle energy of Λ hyperon ε_Λ

$^{27}_{\Lambda}$Mg (AMD)

$$\varepsilon_\Lambda(\beta,\gamma) = E_{\Lambda p}(\beta,\gamma) - E_{core}(\beta,\gamma)$$



- Λ single-particle energy is different in each p orbit with triaxial deformation
- 3 different p-states appear if the core nucleus is triaxially deformed

Production cross section of hypernuclei with HyperAMD

- Strategy
- Current status
- Future plans

Strategy

Production cross section of hypernuclei with AMD wave functions to see effects of various structures

- **In future: (γ, K⁺) reaction**

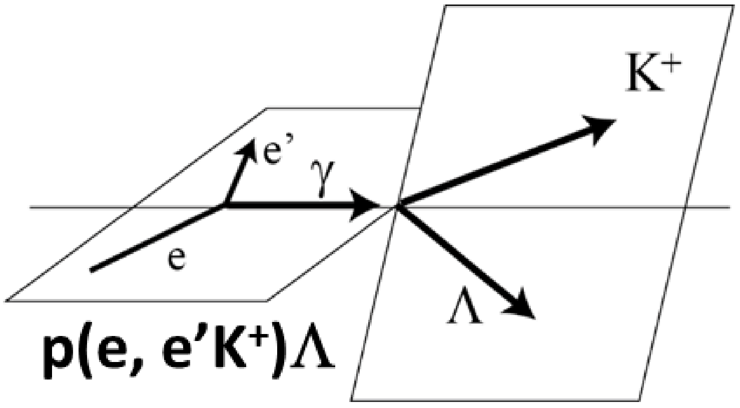
T. Motoba *et al.*, PTP**185**, 224(2010)

$$\frac{d\sigma}{d\Omega}(\theta_K^{\mathrm{Lab}}) = \frac{s p_K^2 E_K E_H}{p_K(E_H+E_K) - E_\gamma E_K \cos\theta_K^{\mathrm{Lab}}} \sum_{M_f} R(fi;M_f),$$

$$R(fi;M_f) = \frac{1}{2J_i+1}\sum_{M_i} \Psi_{\mathrm{GCM}}^{J_f\pi M_f} |\langle \underline{\Psi_{\mathrm{GCM}}^{J_f\pi M_f}} | O | \underline{\Psi_{\mathrm{GCM}}^{J_i\pi M_i}} \rangle|^2$$

AMD + GCM wave functions

Various structure



$$O = \int \underline{d^3 r \chi_K^{(-)*}(\mathbf{p},\xi\mathbf{r})\chi_K^{(+)}(\mathbf{k},\mathbf{r})} \sum_{j=1}^{A} V_-^{(j)} \delta(\mathbf{r}-\eta\mathbf{r}_j) \underline{\langle \mathbf{k}-\mathbf{p},\mathbf{p}|t|\mathbf{k},0\rangle}$$

Plane wave

→ Distorted wave

Elementary amplitude

- **Current status: PWIA based on effective nucleon number approach**

Current Status

◆Effective nucleon number approach using $(d\sigma/d\Omega)_{elem}$

$$\frac{d\sigma}{d\Omega} = \beta \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{elem}} N_{eff}$$

β: kinematic factor

$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{elem}}$: elementary cross section

N_{eff}: effective nucleon number including structure information

$$N_{eff} = \frac{1}{2J_i+1} \sum_{M_1,M_2} \left| \left\langle \Phi_{M_2}^{J_f \pi_f} \middle| \hat{O} \middle| \Phi_{M_1}^{J_i \pi_i} \right\rangle \right|^2$$

AMD + GCM wave functions

Various structure

$$\hat{O}_{NF} = \int d^3x \sum_{j=1}^{A} e^{\vec{q}\cdot\vec{x}/\hbar} \delta(\vec{x} - \vec{r}_j) \hat{V}_j$$

$$\hat{O}_{SF} = \int d^3x \sum_{j=1}^{A} e^{\vec{q}\cdot\vec{x}/\hbar} \delta(\vec{x} - \vec{r}_j) \vec{n} \cdot \vec{\sigma}_j \hat{V}_j$$

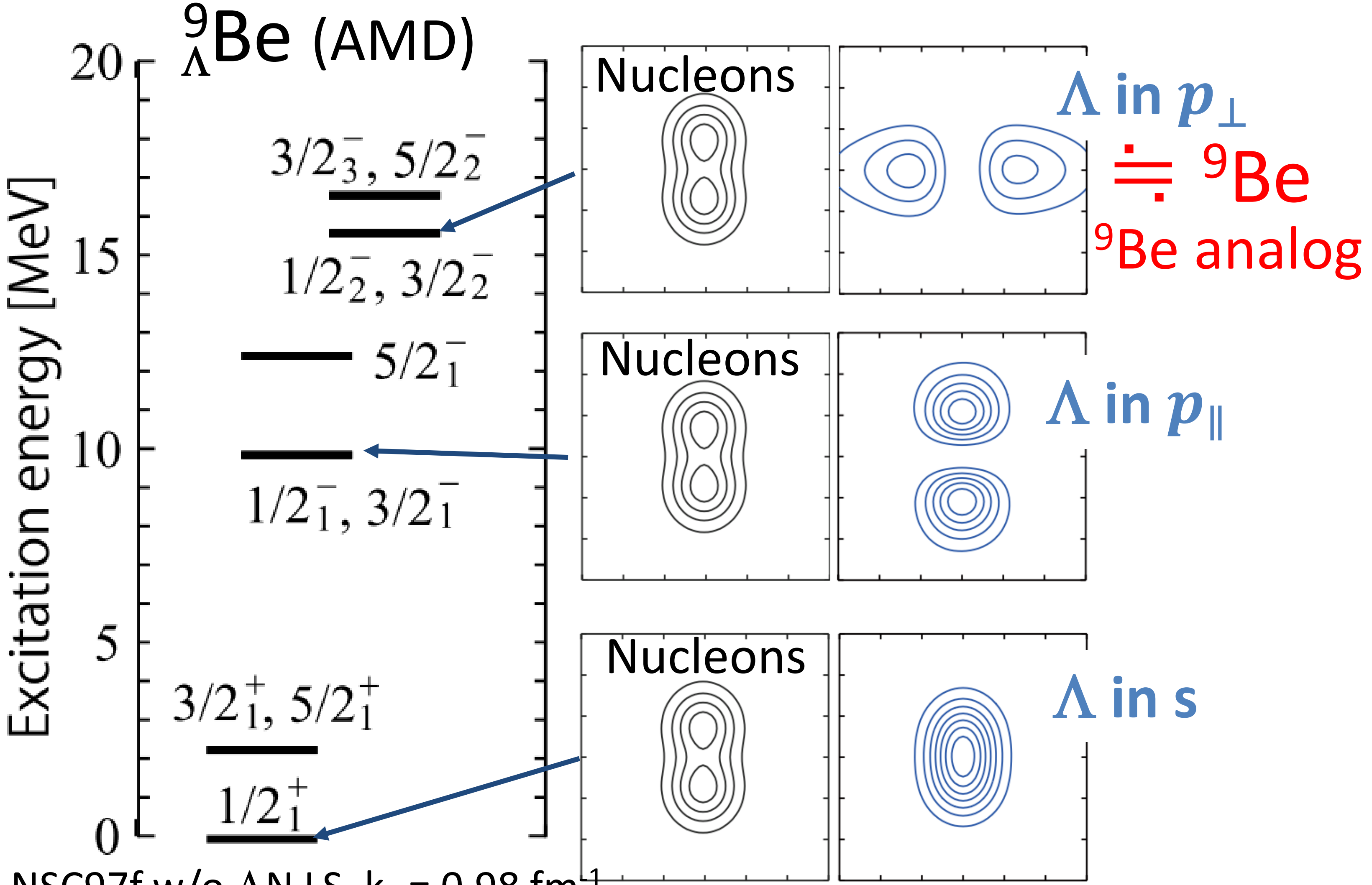
Applicable to (K^-, π^-) & (π^+, K^+) for Λ hypernuclei, (K^-, K^+) for Ξ hypernuclei, … etc.

Current Status

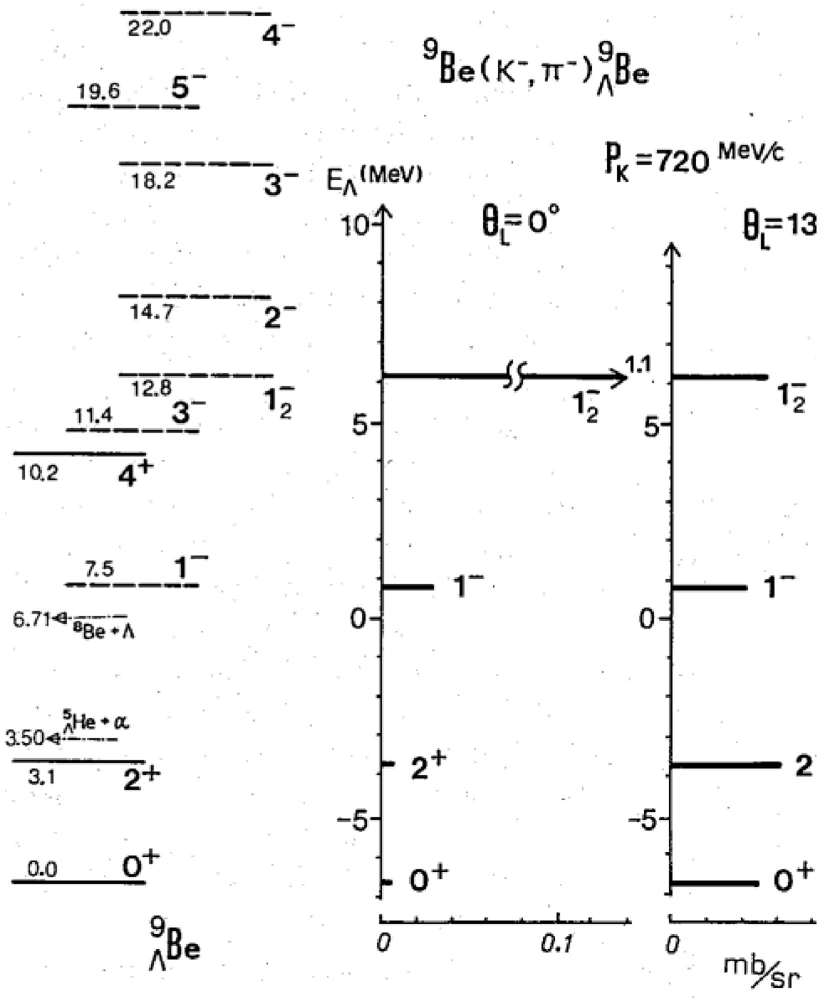
◆Application to ^{9}Be(K^{-}, π^{-})$^{9}_{\Lambda}$Be

cf. $\alpha + \alpha + \Lambda$ cluster model calc

Motoba, et al., PTP71, 222(1984)

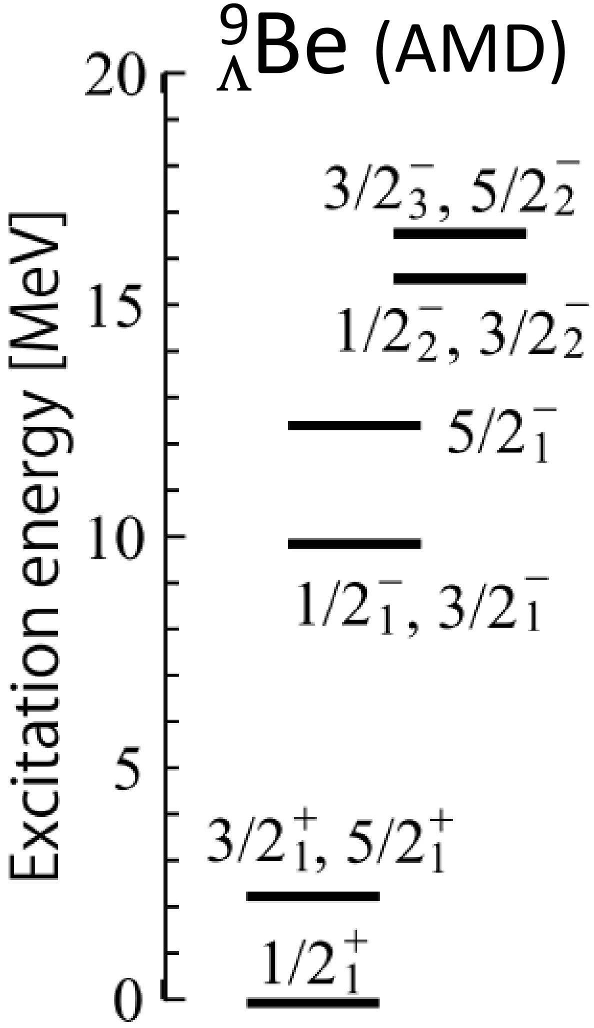


NSC97f w/o ΛN LS, k_F = 0.98 fm^{-1}

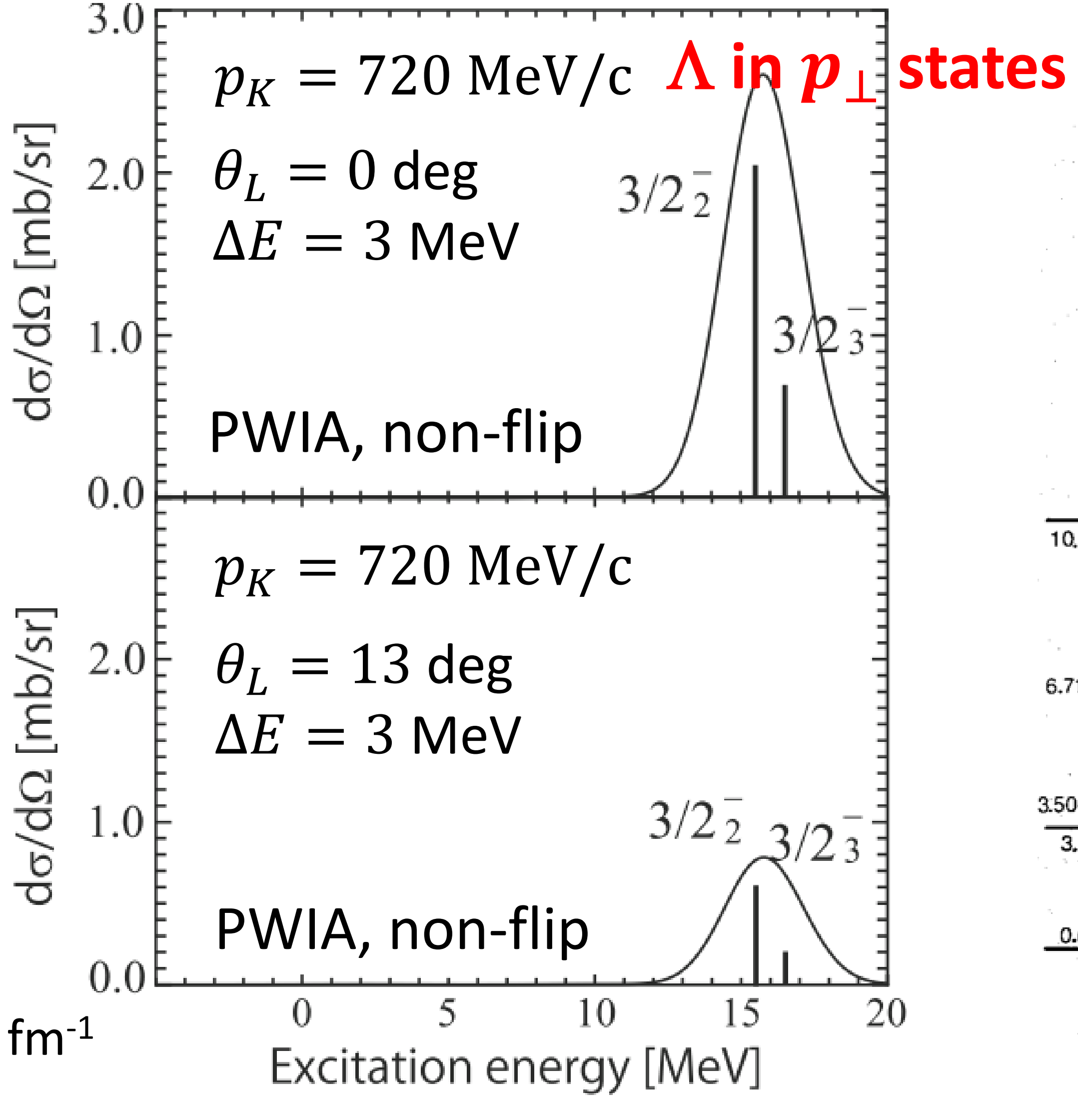


Current Status

◆Application to ^{9}Be(K^{-}, π^{-})$^{9}_{\Lambda}$Be

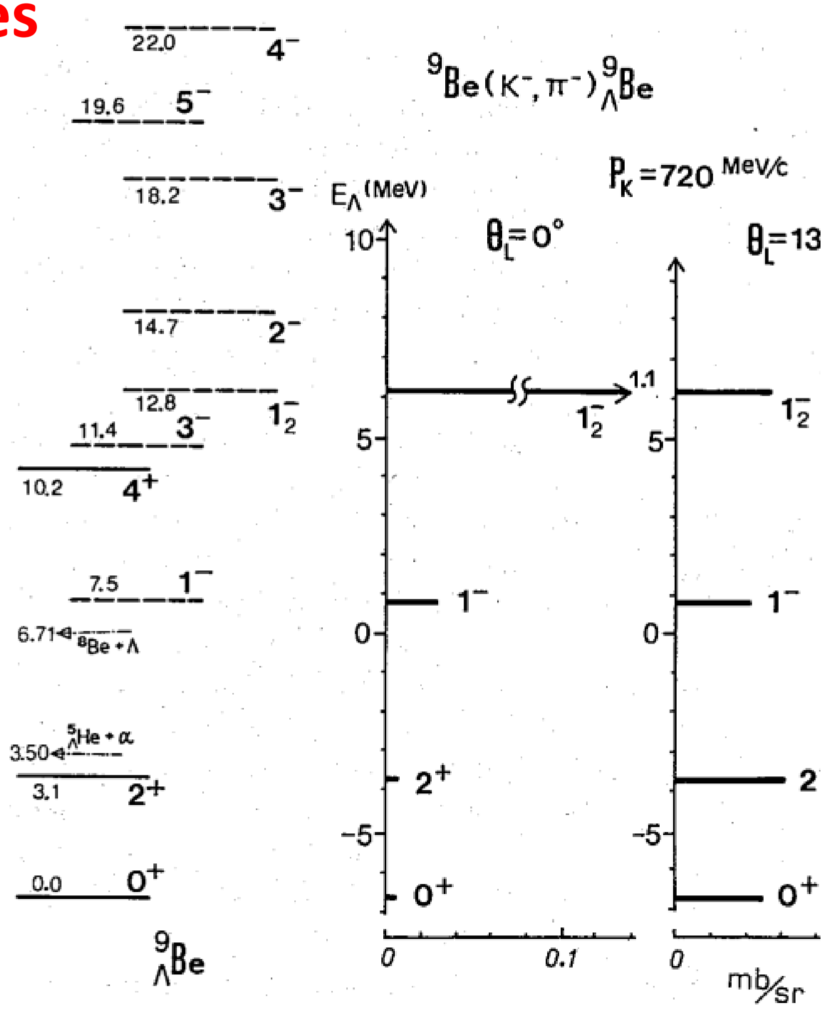


NSC97f w/o ΛN LS, k_F = 0.98 fm^{-1}



cf. α + α + Λ cluster model calc

Motoba, et al., PTP71, 222(1984)

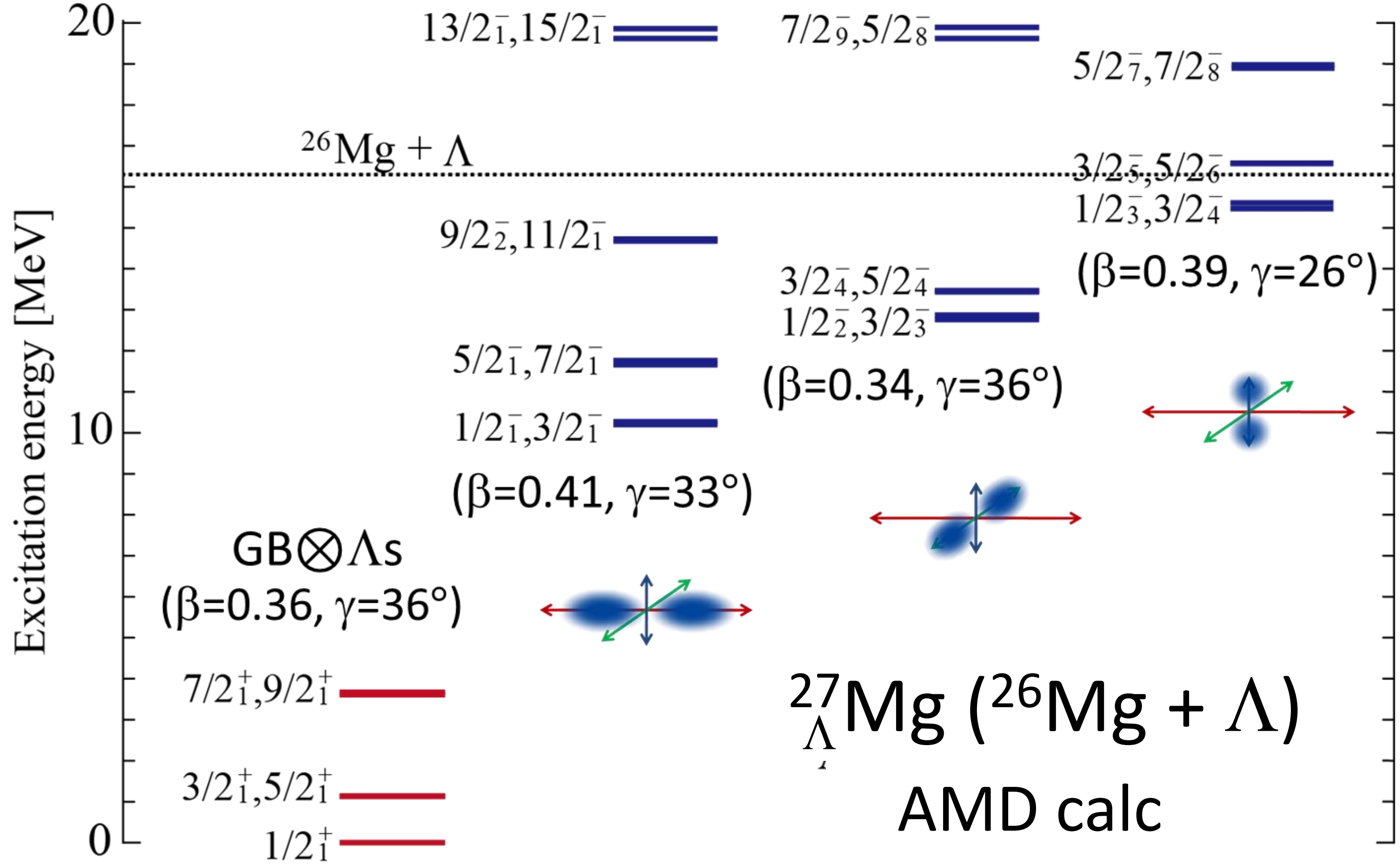


Future plans

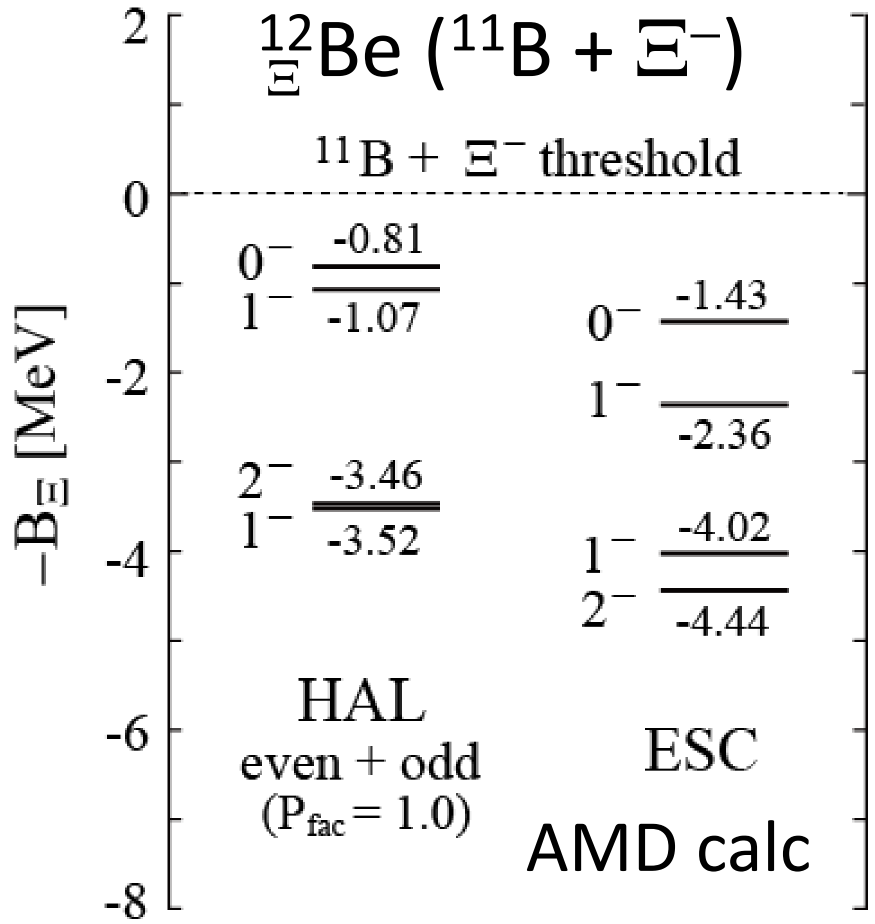
◆Application to (π^+, K^+), (K^-, K^+), and (γ, K^+) reactions

For example,

"How 3 different p-states appear?"



Ξ hypernuclei



Future plans

◆Information of elementary process in *sd*/*pf*-shell hypernuclei

Model dep. of elementary amplitude can affect peak structure

$$O = \int d^3r \chi_K^{(-)*}(\mathbf{p}, \xi\mathbf{r}) \chi_K^{(+)}(\mathbf{k}, \mathbf{r}) \sum_{j=1}^{A} V_-^{(j)} \delta(\mathbf{r} - \eta\mathbf{r}_j) \underline{\langle \mathbf{k} - \mathbf{p}, \mathbf{p} | t | \mathbf{k}, 0 \rangle}$$

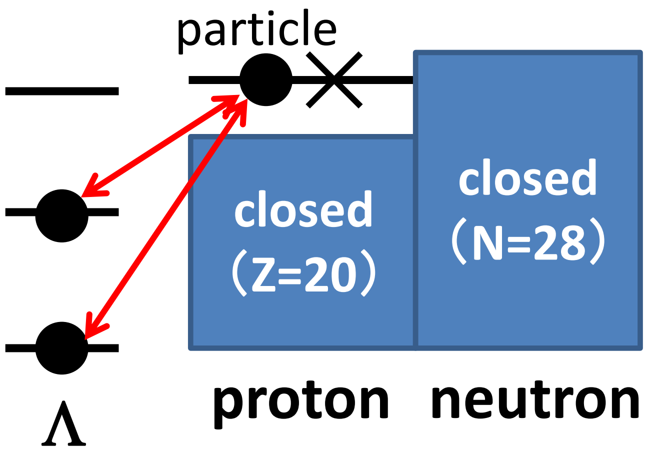
Sotona, Frullani, PTP**117**,151(1994)
Motoba, et al, PTPS**185**, 224(2010)

Input: elementary amplitude by theoretical model

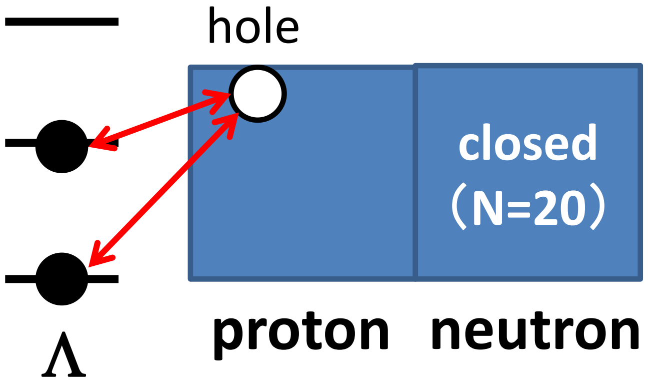
- In sd/pf shell hypernuclei, a proton in higher orbit is converted to hyperon
- Angular momentum dep. of elementary amplitude can affect peak structure

Example:

$^{50}Ti(e,e'K^+)^{50}_{\Lambda}Sc$



$^{40}Ca(e,e'K^+)^{40}_{\Lambda}K$



Summary

◆Hypernuclear deformation

- **What is expected:**
  - Deformation change by Λ particle
  - Difference of B_Λ depending on deformation
  - Coupling of Λ to deformed nuclei

Today's topic: *p*-states in triaxially deformed Mg hypernuclei

→ possibility to use Λ as a probe of deformation

◆Hypernuclear production with HyperAMD

- **Current status:**

  Effective nucleon number approach with PWIA using elem. cross section

- **Future plans: applying to (K^-, K^+), (γ, K^+) reactions**
  - **Unique structure of hypernuclei, elementary amplitudes, … etc.**