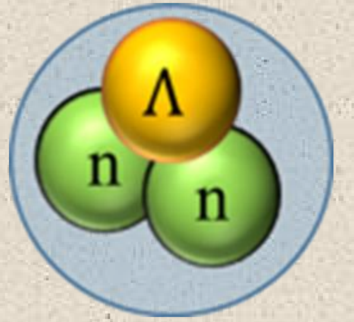
Study of the Λn final state interaction from $^{3}H(e,e'K^{+})X$ spectroscopy

K. Itabashi
for the JLab Hypernuclear Collaboration

Contents

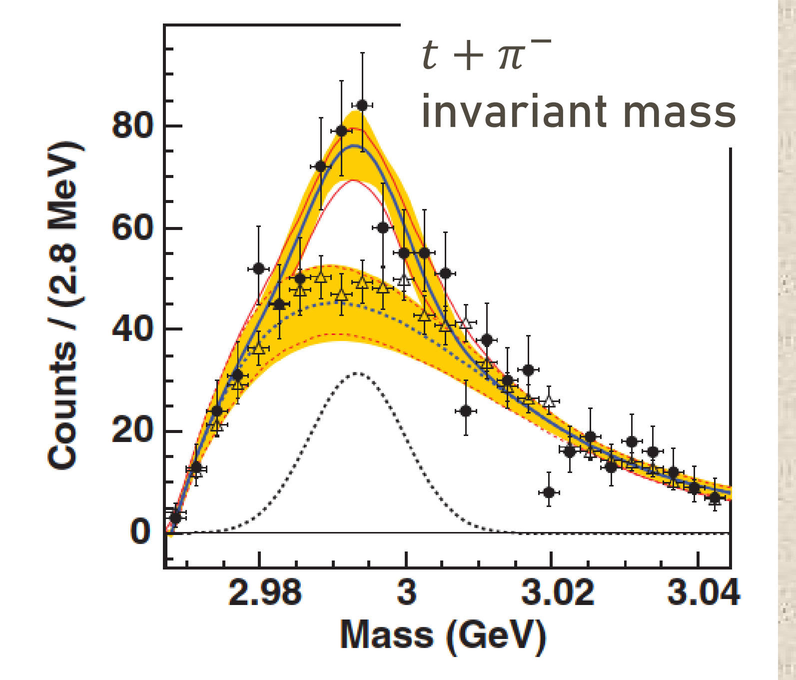
- Physics motivation
- Experimental setup
- Missing mass spectrum in the $^{3}\mathrm{H}(\mathrm{e},\mathrm{e}'K^{+})\mathrm{X}$ reaction
  - Study of the Λn final state interaction

The $nn\Lambda$ state problem



Experimental suggestion

HypHI Collaboration at GSI reported structure that may be interpreted as a bound state of $nn\Lambda$ system.



C. Rappold *et al.*, (HypHI Collaboration) Phys. Rev. C 88 041001 (2013)

Theoretical suggestion

- Theoretical calculation with Gaussian expansion method
  Ref.) E. Hiyama et al., Phys. Rev. C 89, 061302 (2014).
  Bound state of the $nn\Lambda$ is not realistic
- Faddeev calculation with S-wave separable potentials
  Ref.) I.R. Afnan et al., Phys. Rev. C, 92 054608 (2015).
  $nn\Lambda$ could be resonance state when a Λn potential is 5% deeper than Λp potential ($s > 1.05$).

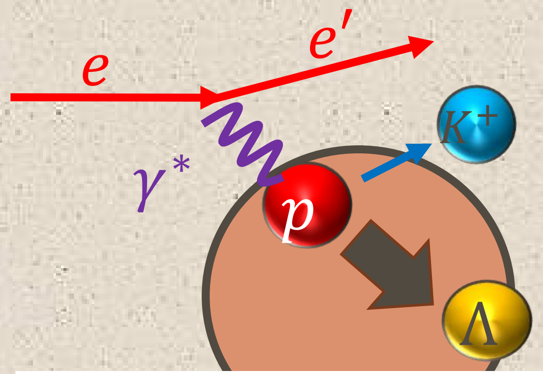
Existence of the $nn\Lambda$ is not established at all
→ Need more precise spectroscopy measurement

Λ hypernucler experiment in the $(e, e'K^+)$ reaction

$(e, e'K^+)$ reaction

$(e \rightarrow e' + \gamma^*)$ to produce Λ in the nucleus.
The missing mass of Λ hypernuclei is

$$M_X = \sqrt{(E_e + m_T - E_{e'} - E_K)^2 - (\overrightarrow{p_e} - \overrightarrow{p_{e'}} - \overrightarrow{p_K})^2}$$



$^{A}Z(e, e'K^+)^{A}_{\Lambda}(Z-1)$

An experiment in the $(e, e'K^+)$ reaction can achievable high energy resolution (a few MeV FWHM) and precision (a few hundreds keV) due to use

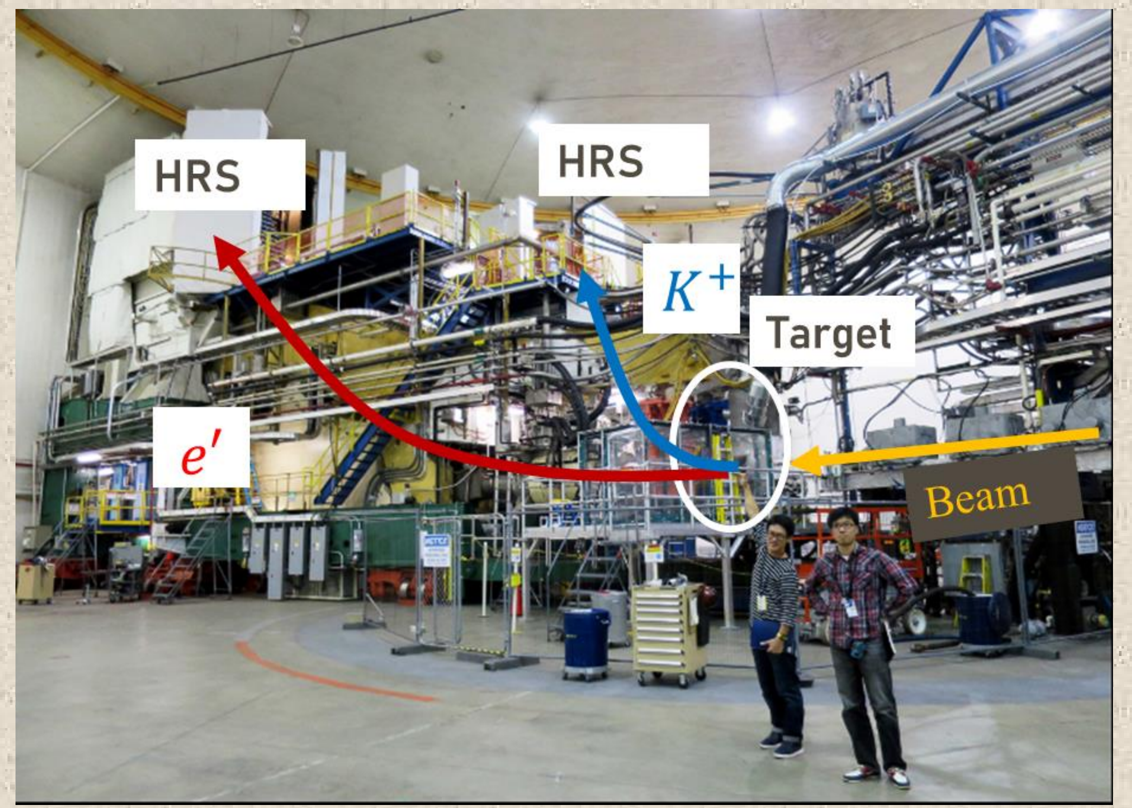
- primary beam with small beam energy spread
- energy calibration with known mases of Λ and Σ^0 in the $p(e, e'K^+)\Lambda/\Sigma^0$ reaction

To search for $nn\Lambda$ with high energy resolution and precision, we performed $nn\Lambda$ experiment at JLab with the $^3\mathrm{H}(e, e'K^+)X$ reaction.

$nn\Lambda$ search experiment (E12-17-003) at JLab

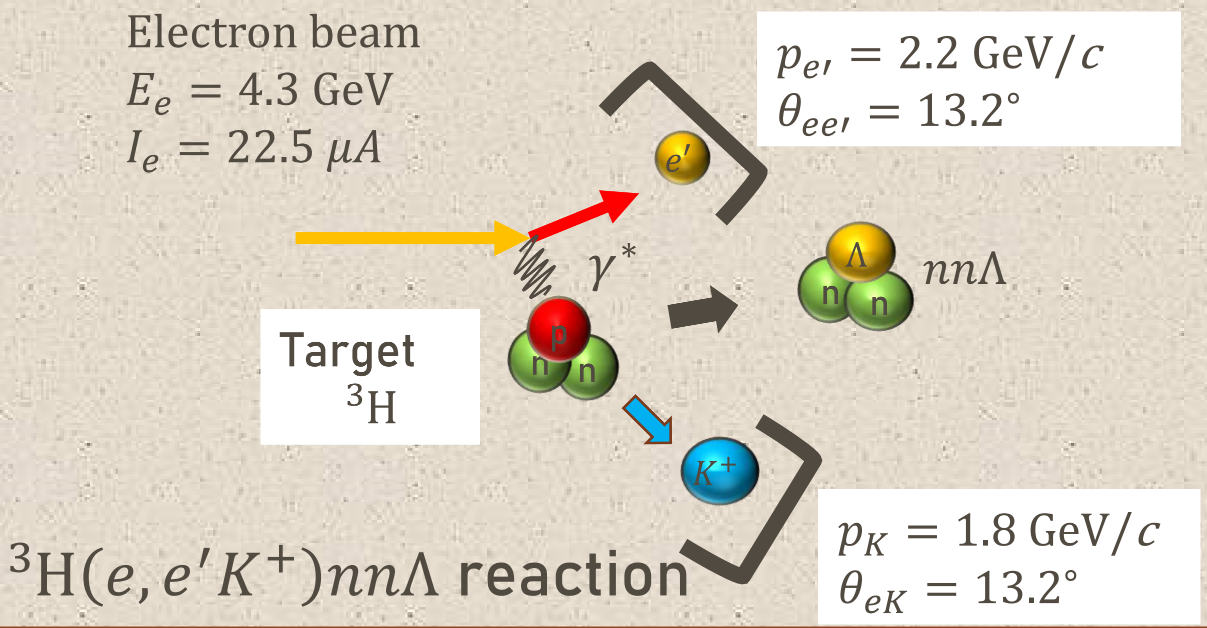
The $nn\Lambda$ search experiment (E12-17-003) was performed at JLab in 2018.

- Two high resolution spectrometers (HRSs) $(\Delta p/p \sim 2.0 \times 10^{-4})$
- Tritium gas target $(84.8\ \mathrm{mg/cm^2})$



The missing mass of the $nn\Lambda$ was obtained by measuring momenta of K^+ and e' with the HRSs

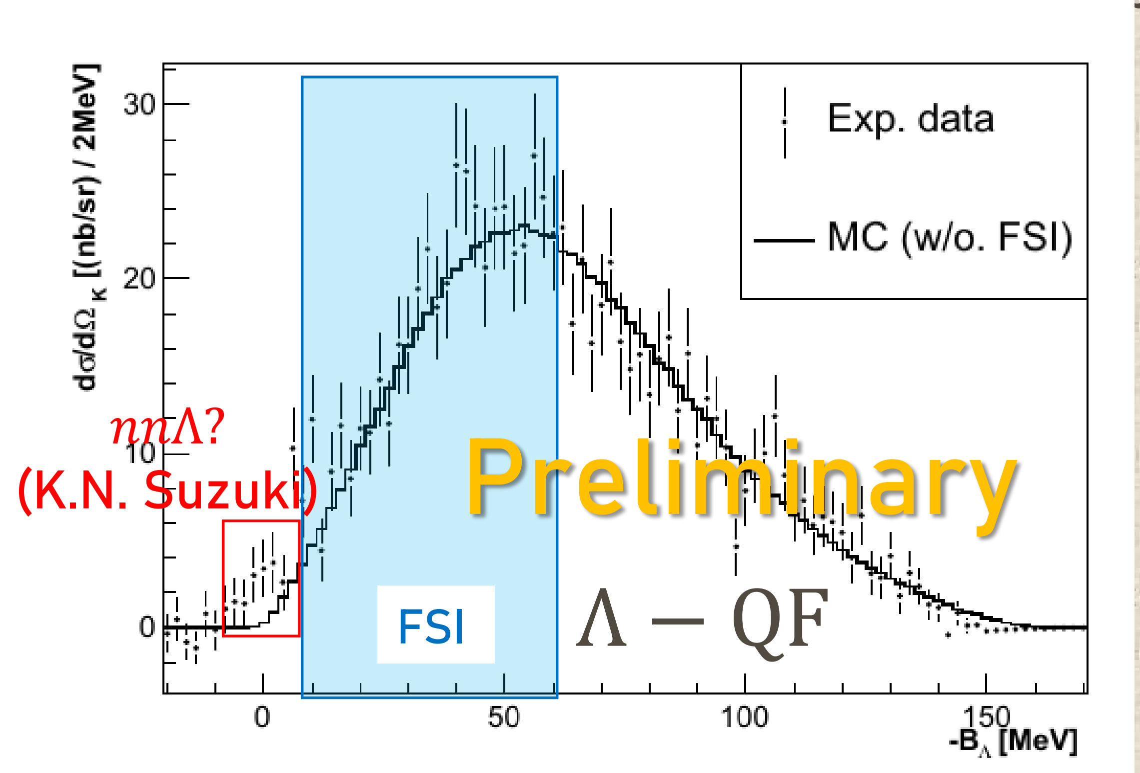
$$M_X = \sqrt{(E_e + m_T - E_{e'} - E_K)^2 - (\vec{p_e} - \vec{p_{e'}} - \vec{p_K})^2}$$



Missing mass spectrum in the $^3\mathrm{H}(\mathrm{e},\mathrm{e}'\mathrm{K}^+)\mathrm{X}$

Distribution of the Λ-QF production was estimated by Monte Carlo simulation (SIMC)

$^3\mathrm{H}(\mathrm{e},\mathrm{e}'\mathrm{K}^+)\mathrm{X}$ spectrum



The MC distribution was generated by SIMC with physics effects as

- Fermi momentum of proton in $^3\mathrm{H}$
  Ref.) R. B. Wiringa Phys. Rev. C 43, 1585 (1991).
- Kaon decay effect
- Radiative effects

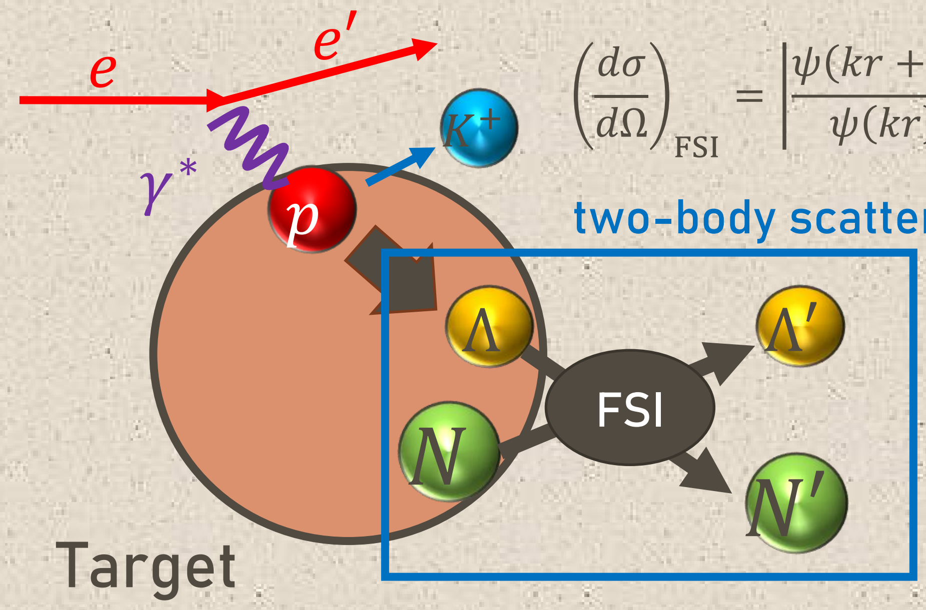
From missing mass spectrum in the $^3\mathrm{H}(\mathrm{e},\mathrm{e}'\mathrm{K}^+)\mathrm{X}$ reaction,

the following physics have been studied

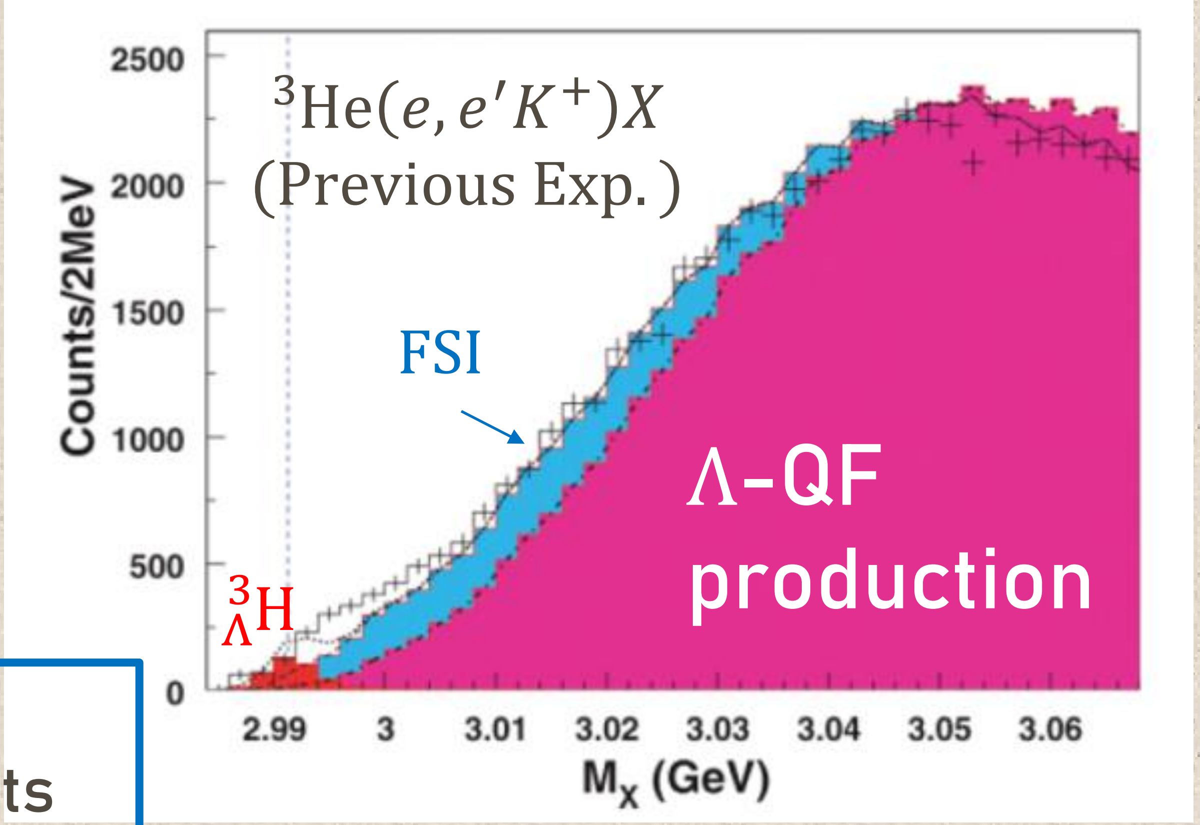
1. Upper limit of the $nn\Lambda$ from an event excess ($-B_\Lambda \sim 0$ MeV)
   Ref.) K.N. Suzuki *et al.*, Prog. of Theo. and Exp. Phys, 2022, 013D01 (2022).
2. Λn final state interaction from the Λ-QF spectrum ($0 \leq -B_\Lambda \leq 60$ MeV)

Previous study of Final State Interaction (FSI)

The recoil Λ interacts with a nucleon within a target system (ΛN scattering)



$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{FSI}} = \left|\frac{\psi(kr+\delta)}{\psi(kr)}\right|^2 \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{w/o\ FSI}}$$



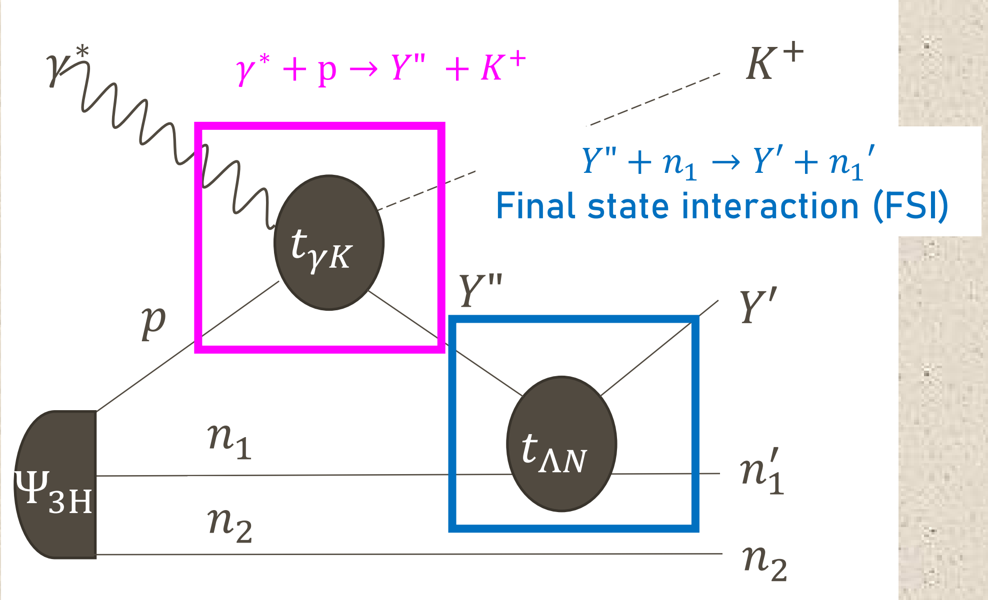
ΛN final state interaction (FSI)

→ Successfully reproduced the excess events

Calculation of Λn final state interaction

Λn final state interaction (FSI) : The recoil Λ interacts with a neutron within nn system.
FSI can be written with influence factor $I(k_{\rm rel})$ as following

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm FSI} = \left|\frac{\psi(kr+\delta)}{\psi(kr)}\right|^2 \left(\frac{d\sigma}{d\Omega}\right)_{\rm w/o\ FSI} = I(k_{rel})\left(\frac{d\sigma}{d\Omega}\right)_{\rm w/o\ FSI} = \frac{1}{|J_l(k_{rel})|^2}\left(\frac{d\sigma}{d\Omega}\right)_{\rm w/o\ FSI}$$



In the ERA ($k\cot\delta = 1/a + 1/2 r_e k^2$), the Jost function is written with potential parameters, scattering length (a) and effective range (r_e) as :

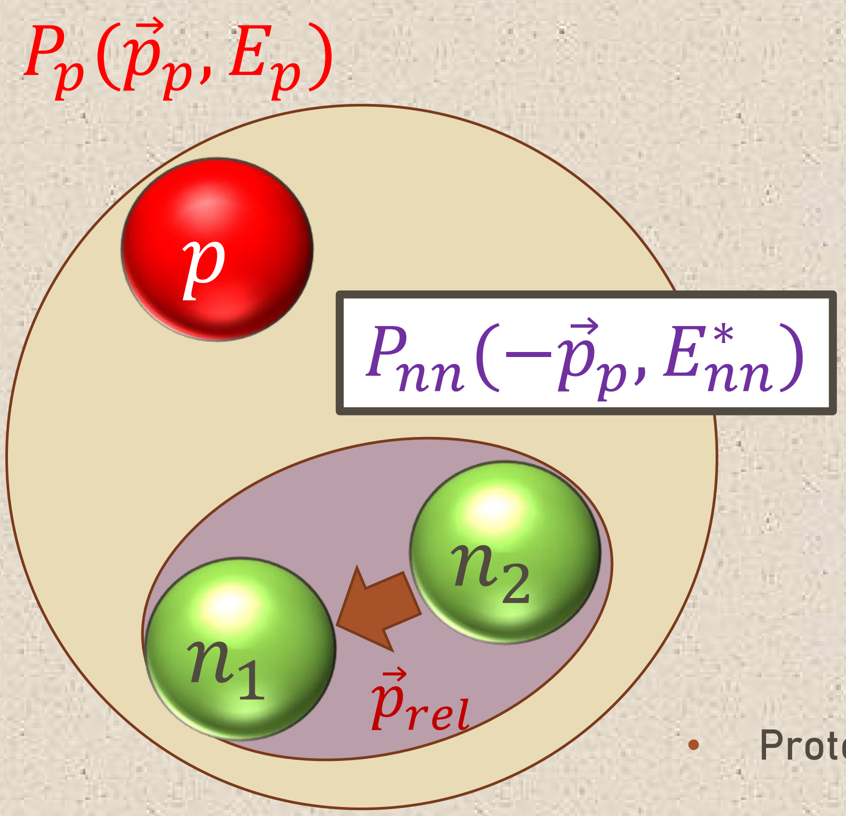
$$J_{l=0}(k_{rel}) = \frac{k_{\rm rel} - i\beta}{k_{\rm rel} - i\alpha}$$

$$\frac{1}{2} r_e(\alpha - \beta) = 1,\ \frac{1}{2} r_e \alpha\beta = -\frac{1}{a}$$

There are three free parameters $(a, r, k_{\rm rel})$

(a, r) : determined by models
Need to calculate k_{rel}

Neutron momentum calculation



Stopped tritium target → $\vec{p}_p + \vec{p}_{n1} + \vec{p}_{n2} = 0$

Relative momentum was defined as $\vec{p}_{rel} = \frac{M_n\vec{p}_{n1} - M_n\vec{p}_{n2}}{2M_n}$

$$\vec{p}_{n1(n2)} = -\frac{1}{2}\vec{p}_p + \vec{p}_{rel}$$

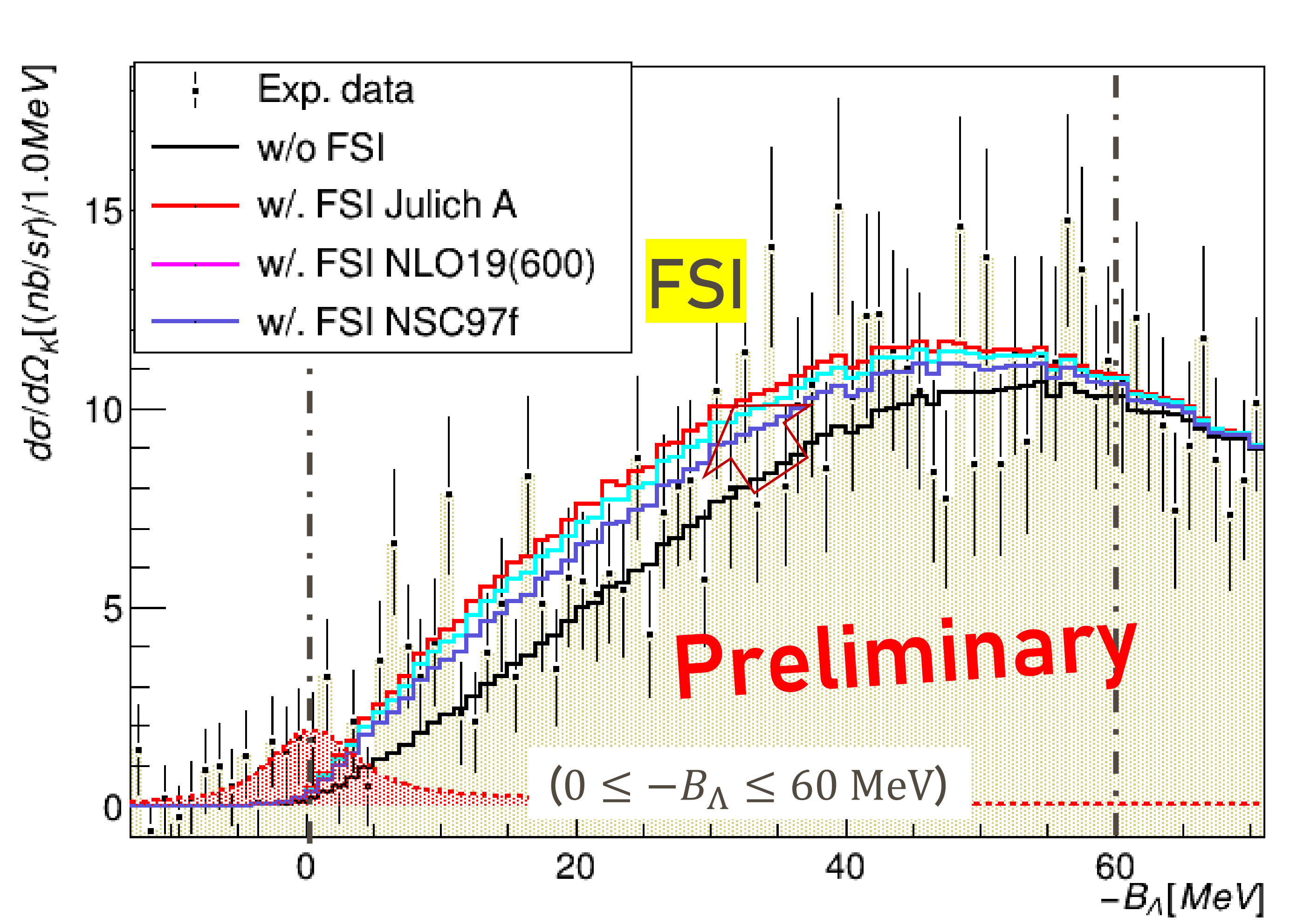
$$|\vec{p}_{n1(n2)}| = \sqrt{|\vec{p}_{rel}|^2 + \frac{|\vec{p}_p|^2}{4} \mp |\vec{p}_p||\vec{p}_{rel}|\cos\theta}$$

θ: angle between proton and relative momentum

- Proton momentum (p_p) : Fermi momentum distribution
  Ref.) R. B. Wiringa Phys. Rev. C 43, 1585 (1991).
- Angle between $\vec{p}_p$ and $\vec{p}_{rel}$ (θ) : Assuming spherical uniform distribution
- Relative momentum $(\vec{p}_{rel})$: Given by an excited energy of nn system (E^*_{nn})

E^*_{nn} was estimated by spectral function of $^3\mathrm{H}$ Ref.) C. Ciofi degli Atti et al., Phys. Rev. C, 21 (1980).

Calculation of the Λn final state interaction



$nn\Lambda$ peak ($-B_\Lambda \sim 0$ MeV):

Breit-Wigner with $(-B_\Lambda, \Gamma) = (0.55, 4.7)$ MeV

Ref.) K.N. Suzuki *et al.,* Prog. of Theo. and Exp. Phys, 2022, 013D01 (2022).

Scaling factors $(w_{FSI}, w_{nn\Lambda})$ were determined by chi-square $(0 \leq -B_\Lambda \leq 60$ MeV$)$.

$$\chi^2 = \sum_i^{N_{\text{bin}}} \frac{\left(y_{\text{data}}^i - w_{FSI} \cdot y_{\text{FSI}}^i + w_{nn\Lambda} \cdot y_{nn\Lambda}^i\right)^2}{\sigma_{\text{data}}^i}$$

With FSI : Succeeded in producing a structure $(0 \leq -B_\Lambda \leq 60$ MeV$)$

Model difference : Small

- Not enough statistic to identify the model

Search for best (a, r) parameters with chi-square

Λn FSI : calculated by Jost function with the (a, r) potential parameters

- Search for the best (a, r) parameters by changing them

There are four Λn potential parameters (a_s, r_s, a_t, r_t)

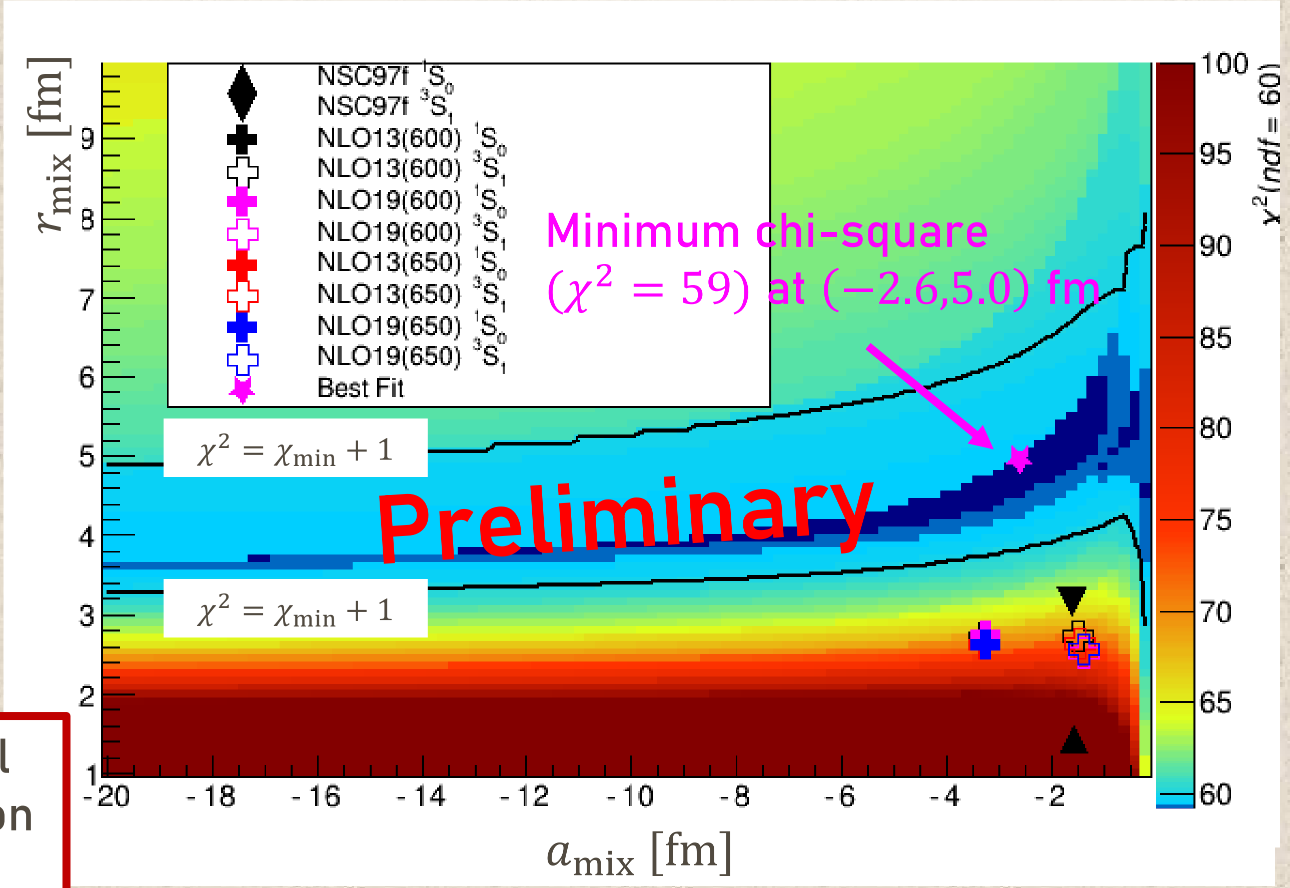
$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{FSI}} = \left(\left|\frac{1}{\mathrm{J_s(k_{rel})}}\right|^2 + 3\left|\frac{1}{\mathrm{J_t(k_{rel})}}\right|^2\right)\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{w/o\ FSI}}$$

In this study, two potential parameters $(a_{\mathrm{mix}}, r_{\mathrm{mix}})$ were used (mixed spin state of a and r)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{FSI}} = \left(\left|\frac{1}{\mathrm{J_{mix}(k_{rel})}}\right|^2\right)\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{w/o\ FSI}}$$

Assuming $a_{\mathrm{mix}} = -2.6$ fm

(Preliminary) $r_{\mathrm{mix}} = 5.0^{+1.3}_{-1.2}$ (stat.) fm



$(a_{\mathrm{mix}}, r_{\mathrm{mix}})$ is not directly comparable with the theoretical models. → I will consult with theorists and give restriction on the Λn potential parameters (a_s, r_s, a_t, r_t) in this study.

Conclusion

Study of the Λn potential dependence (preliminary)

- Fitting by chi-square $(0 \leq -B_\Lambda \leq 60\ \mathrm{MeV})$
- NSC97f is the smallest chi-square in seven potential models

Λn Potential	Reduced chi-square (χ^2/ndf)
w/o FSI (w/o nnL peak)	1.24
w/o FSI	1.09
Jülich A	1.40
Jülich B	1.15
NSC97f	1.05
NLO13(600)	1.16
NLO13(650)	1.17
NLO19(600)	1.22
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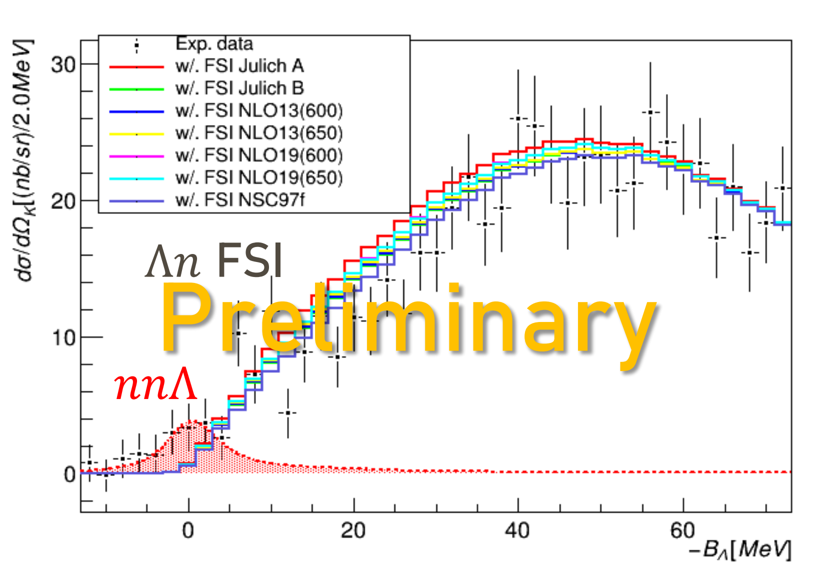
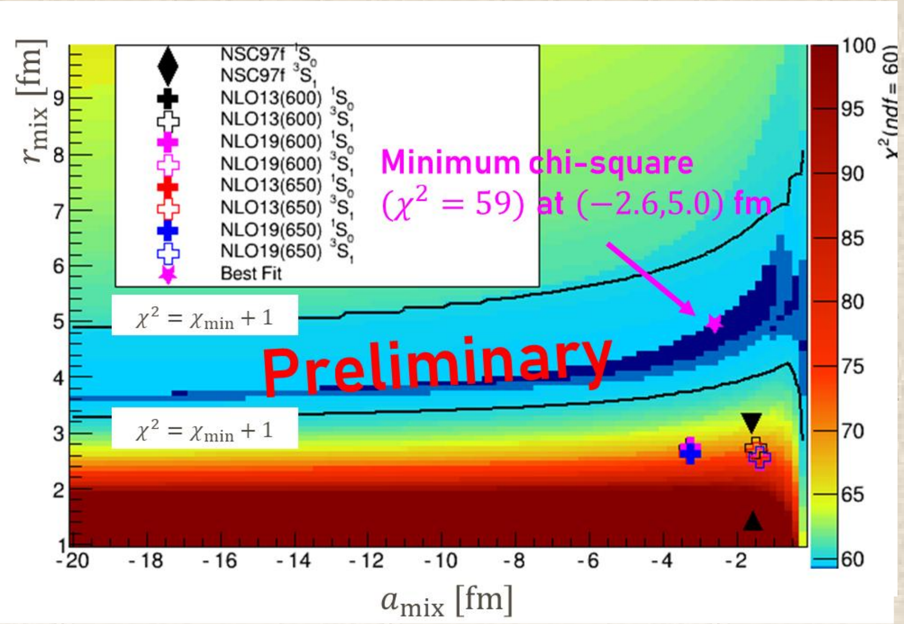
Preliminary



Search for the best fit of Λn FSI (preliminary)

- Minimum chi-square $(\chi^2 = 59)$ at $(-2.6, 5.0)$ fm
- $a_{\mathrm{mix}} = -2.6$ fm is comparable with the Λn potential models
- The effective range (r_{mix}) can be limited for a given a_{mix}.



$(a_{\mathrm{mix}}, r_{\mathrm{mix}})$ is not directly comparable with the theoretical models.
→ I will consult with theorists and give restriction on the Λn potential parameters (a_s, r_s, a_t, r_t) in this study.

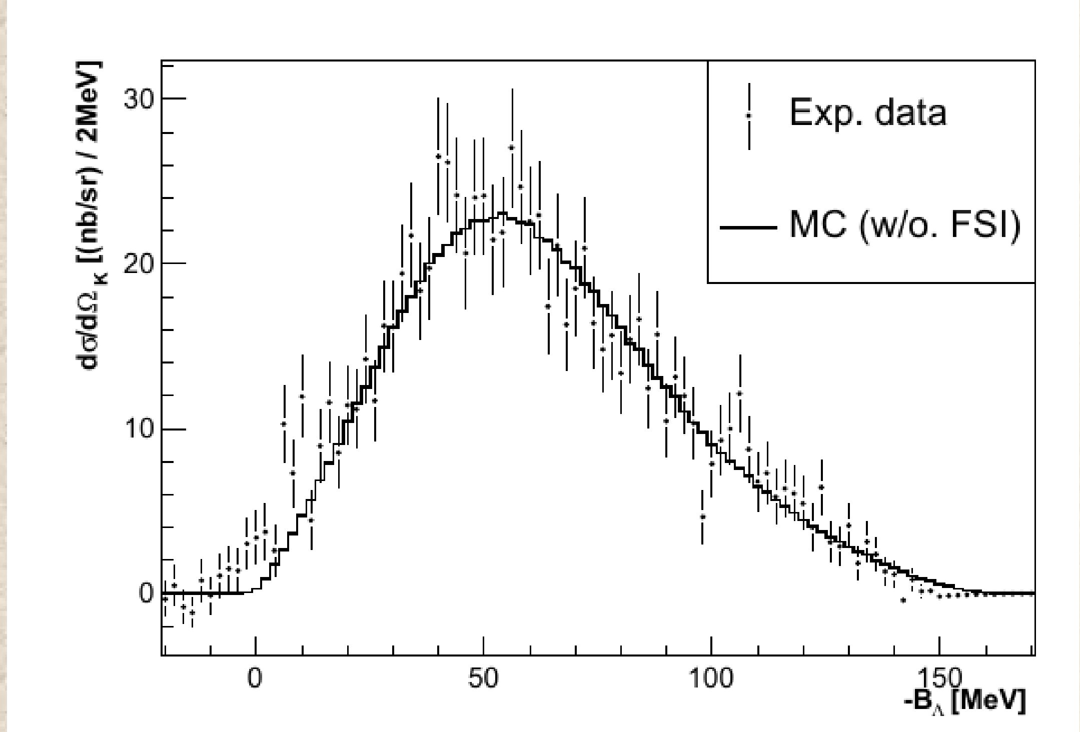
SUMMARY

- The information of the ΛN interaction can be obtained by Λ hypernuclear spectroscopy.
- The $nn\Lambda$ search experiment was performed in 2018 at JLab.
- Λn FSI was studied by fitting the Λ-QF distribution in the $^3\mathrm{H}(e, e'K^+)\mathrm{X}$ reaction.
  - Λn FSI was calculated with Jost function in the ERA
  - NSC97f got smallest chi-square in the seven potential models
  - The effective range (r_{mix}) were limited for a given $\mathrm{a}_{\mathrm{mix}}$.
  - Assuming $a = -2.6$ fm, the effective range was obtained as $r_{\mathrm{mix}} = 5.0^{+1.3}_{-1.2}\,(\mathrm{stat.})$ fm (preliminary)

Backup

Monte Carlo Simulation (SIMC)

Distribution of the Λ-QF production was estimated by Monte Carlo simulation (SIMC)



The MC distribution was generated by SIMC with physics effects as

- Fermi momentum of proton in $^3\mathrm{H}$

  Ref.) R. B. Wiringa Phys. Rev. C 43, 1585 (1991).
- Kaon decay effect
- Radiative effects

$60 < -B_\Lambda < 150$ MeV : Good agreement with data

$0 < -B_\Lambda < 60$ MeV : There are events excesses

$\rightarrow \Lambda n$ FSI effect

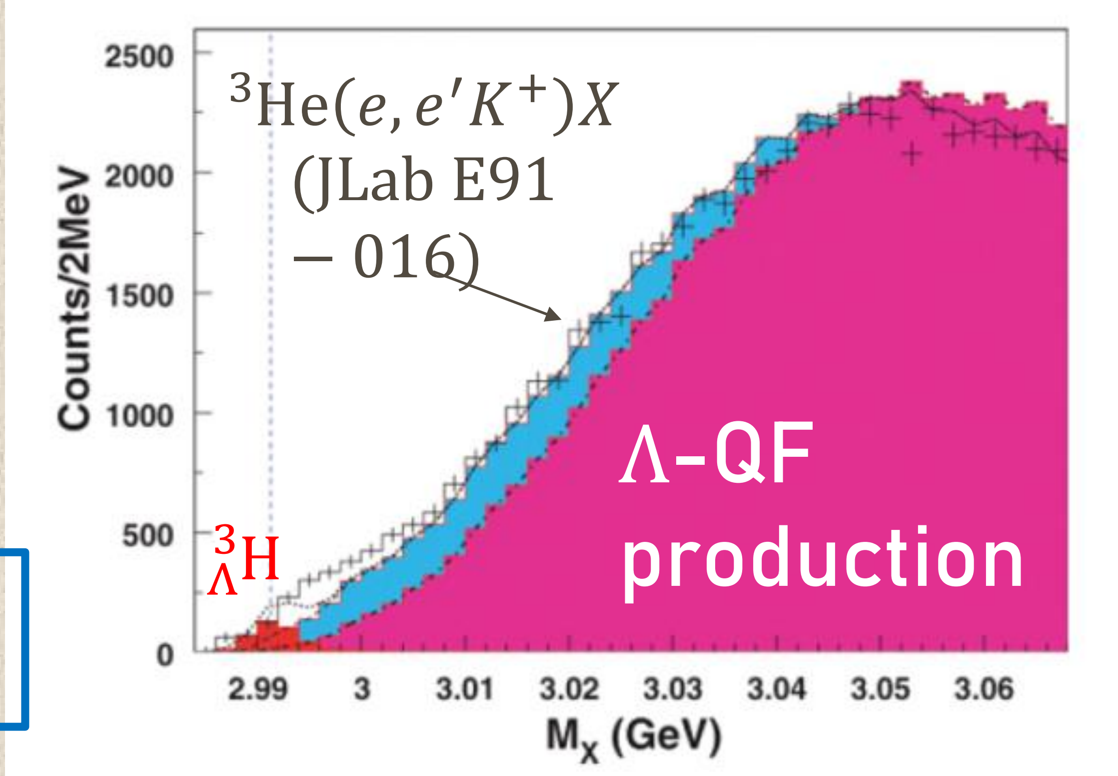
Study of the ΛN interaction from Λ hypernuclei

In the Λ hypernuclear spectroscopy experiment, missing mass of Λ hypernuclei and Λ quasi-free (Λ-QF) productions would be measured.

JLab experiment (E91-016) with $(e, e'K^+)$ reaction

there were excess events $(2.99 < \mathrm{M_X} < 3.05\ \mathrm{GeV})$.

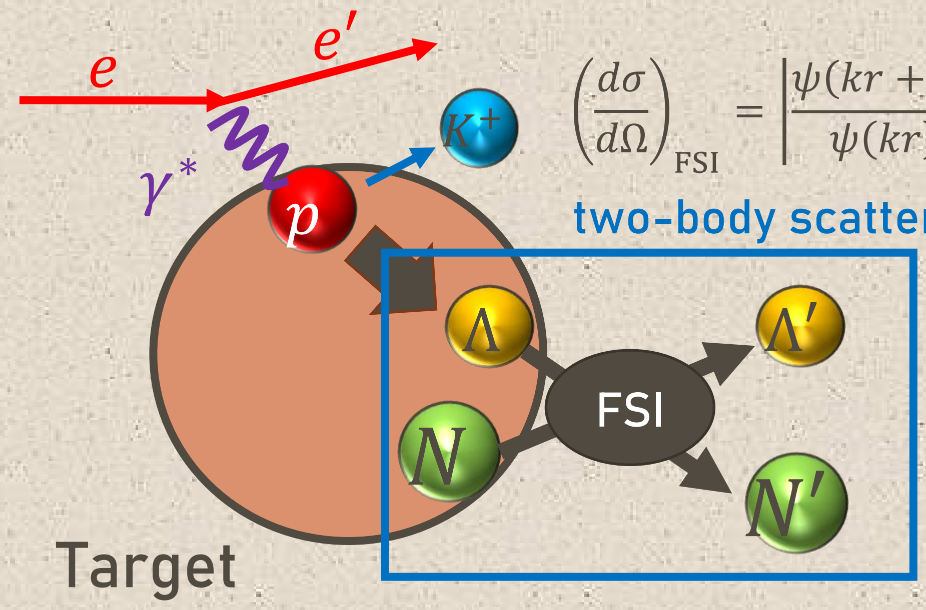
- Black dot points : Experimental data
- magenta histogram : Λ-QF distribution (simulation)



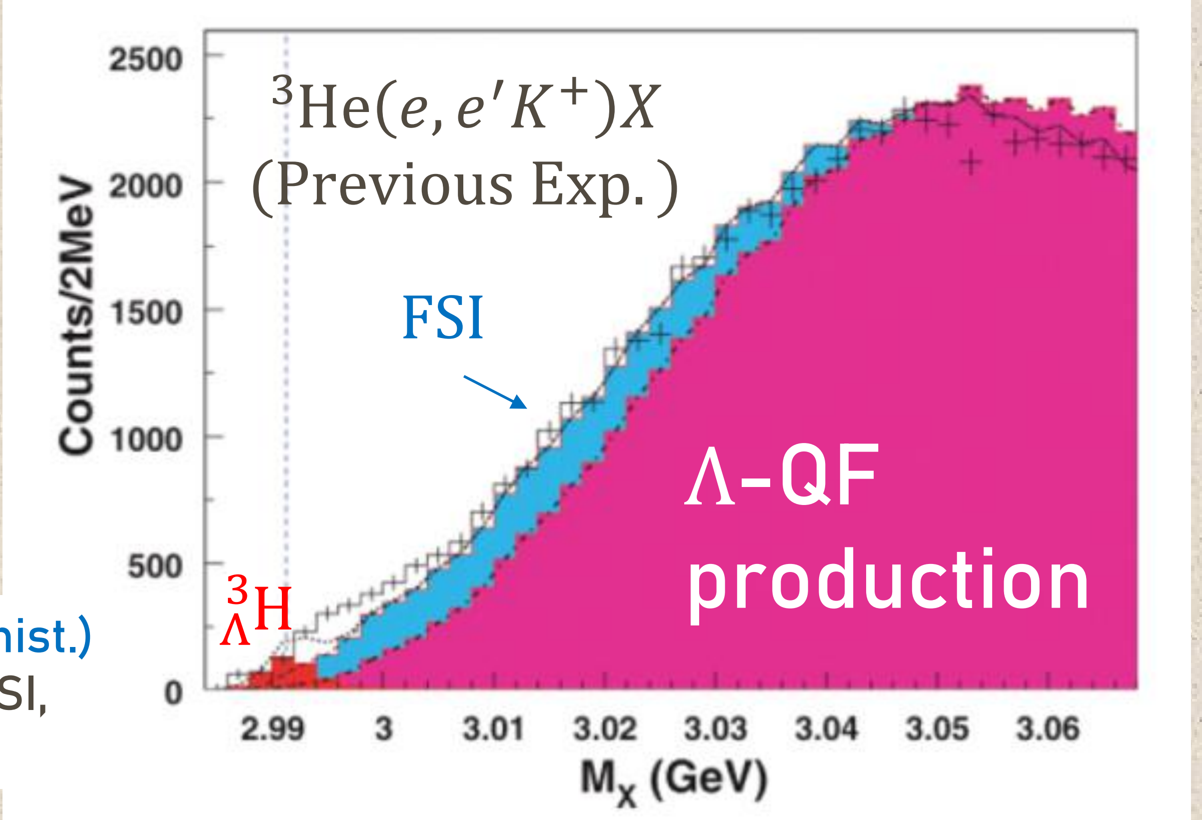
ΛN final state interaction (FSI)
→ Successfully reproduced the excess events

Study of Final State Interaction (FSI)

The recoil Λ interacts with a nucleon within a target system (ΛN scattering)



$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{FSI}} = \left|\frac{\psi(kr+\delta)}{\psi(kr)}\right|^2 \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{w/o\ FSI}}$$

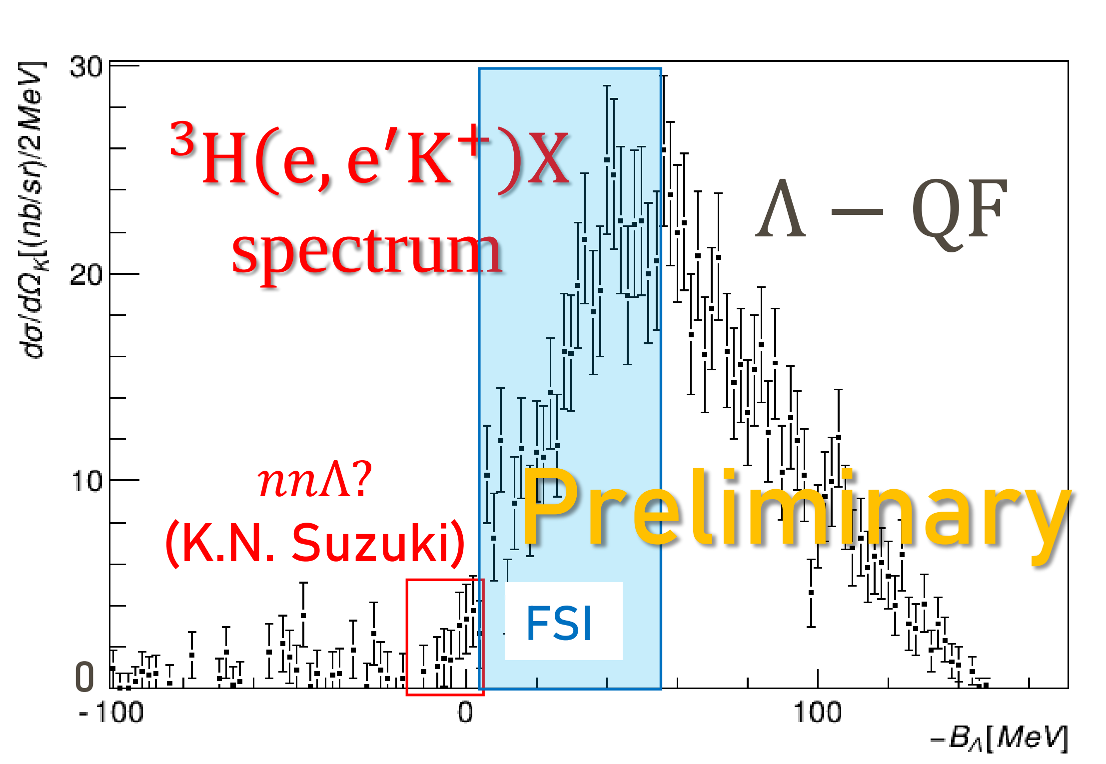


The ΛN scattering in the nucleus make enhancement (blue hist.)
→ By fitting experimental data with Λ-QF distribution with FSI, information of ΛN interaction can be obtained

Missing mass spectrum in the $^3\mathrm{H}(\mathrm{e},\mathrm{e}'\mathrm{K}^+)\mathrm{X}$

The differential cross section of Λ −QF production calculation was calculated by

$$\overline{\left(\frac{d\sigma_{\mathrm{QF}}}{d\Omega_K}\right)} = \frac{1}{N_T}\frac{1}{N_{\gamma^*}}\frac{1}{\varepsilon_{\det}}\sum_{i}^{\mathrm{N_{QF}}}\frac{1}{\varepsilon_K^i(\vec{p}_K^{\,i})\,d\Omega_K(\vec{p}_K^{\,i})}$$

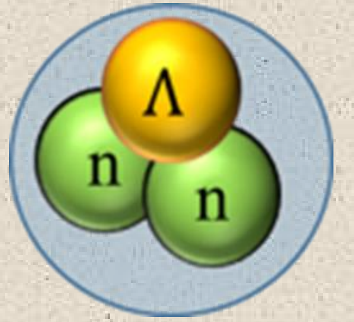


From missing mass spectrum in the $^3\mathrm{H}(\mathrm{e},\mathrm{e}'\mathrm{K}^+)\mathrm{X}$ reaction,

the following physics have been studied

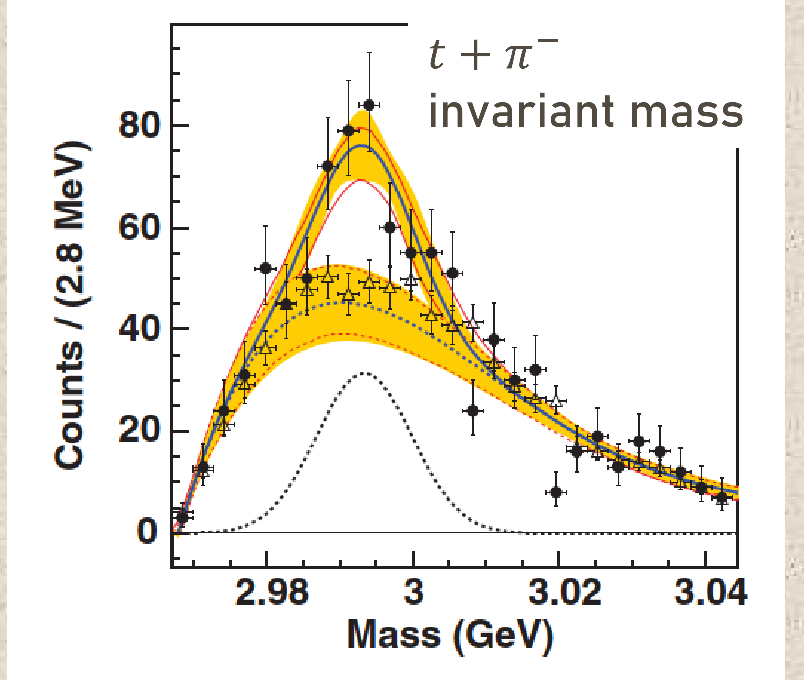
1. Upper limit of the $nn\Lambda$ from an event excess $(-B_\Lambda \sim 0\ \mathrm{MeV})$
   Ref.) K.N. Suzuki *et al.,* Prog. of Theo. and Exp. Phys, 2022, 013D01 (2022).
2. Λn final state interaction from the Λ-QF spectrum $(0 \leq -B_\Lambda \leq 60\ \mathrm{MeV})$

The $nn\Lambda$ state problem



A $nn\Lambda$ is a neutron-Λ system with no charge.

→ Existence of $nn\Lambda$ is not established at all.



C. Rappold *et al.*, (HypHI Collaboration) Phys. Rev. C 88 041001 (2013)

HypHI Collaboration at GSI reported structure that may be interpreted as a bound state of $nn\Lambda$ system.

Experimental data(GSI)

- Invariant mass : $m_{nn\Lambda}$=2994.3±1.1(stat.)±2.2(sys.) MeV/c^2
- Lifetime : $\tau_{nn\Lambda} = 190^{+47}_{-35}$(stat.)±36(sys.) ps

→ Peak events imply bound state of the $nn\Lambda$

However, GSI did not measure enough significance.

$nn\Lambda$ state problem (theoretical discussion)

HypHI collaboration measured the events which indicate bound state of $nn\Lambda$.

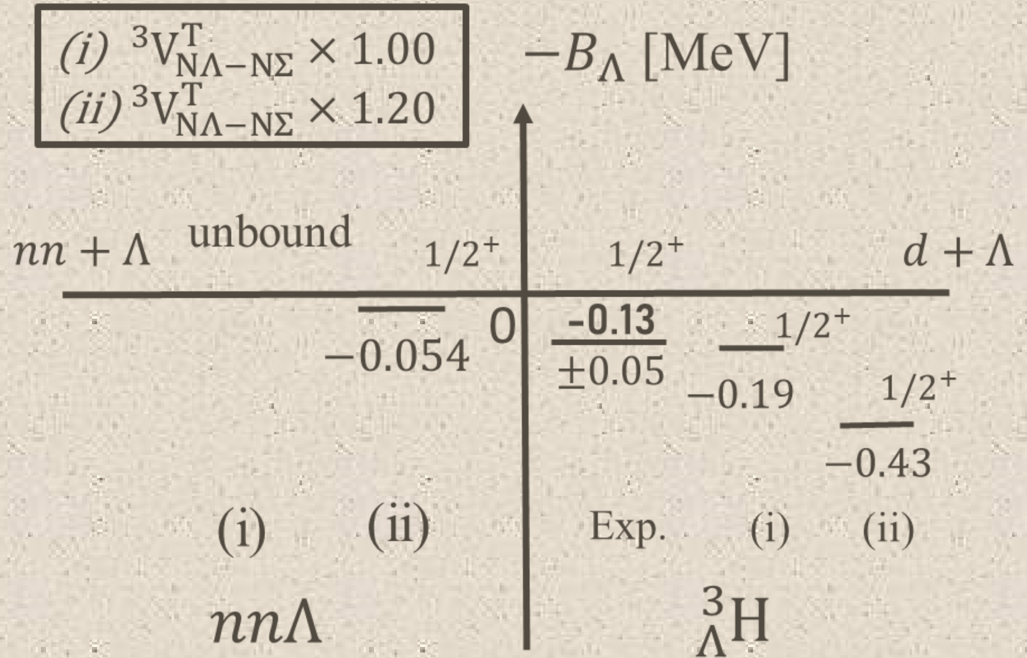
However, theoretical calculations cannot reproduce bound state of $nn\Lambda$.

Theoretical calculation with Gaussian expansion method

ΛN interaction : NSC97f model including $\Lambda N - \Sigma N$ coupling effect

The binding energies in ${}^{3,4}_{\Lambda}\mathrm{H}$ and ${}^{4}_{\Lambda}\mathrm{He}$ are reproduced

E. Hiyama et al., Phys. Rev. C 89, 061302 (2014).

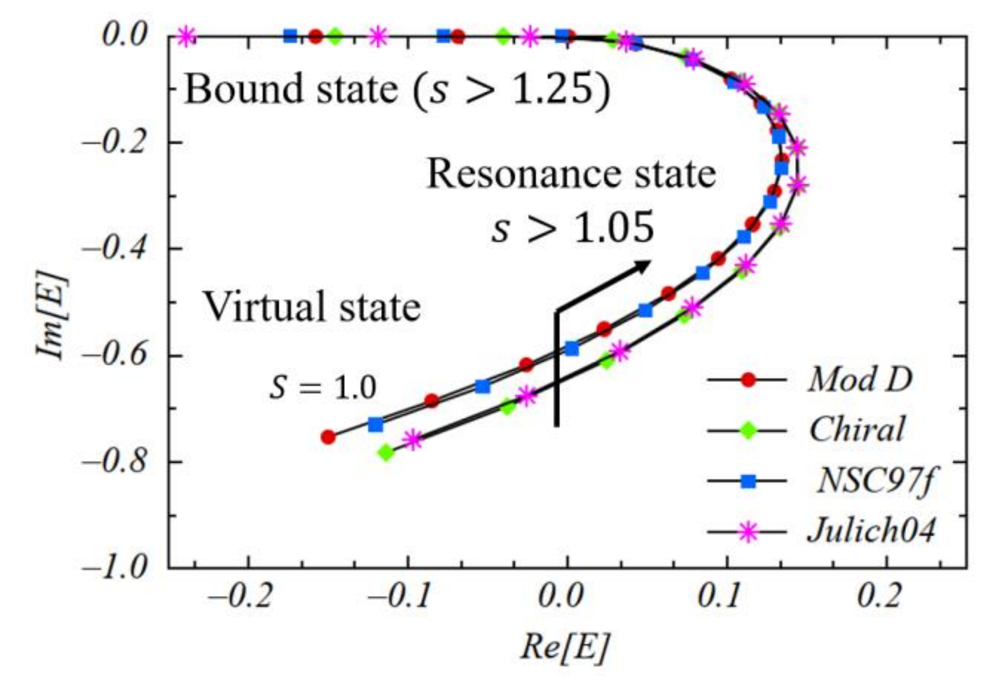


if ${}^{3}V^{T}_{N\Lambda-N\Sigma}$ parameter increase 20%,
$nn\Lambda$ will be bound, but ${}^{3}_{\Lambda}\mathrm{H}$ will be over bound
→ The bound state of $nn\Lambda$ is unrealistic

$nn\Lambda$ state problem (theoretical discusion)

Faddeev calculation with S-wave separable potentials suggested that $nn\Lambda$ could be resonance state

when a Λn potential is 5% deeper than Λp potential ($s > 1.05$).



I.R. Afnan et al., Phys. Rev. C, 92 054608 (2015).

$s = 1$ assuming charge symmetry$\left(V_{\Lambda n} = 1.0 \times V_{\Lambda p}\right)$

Λp interaction has uncertainly

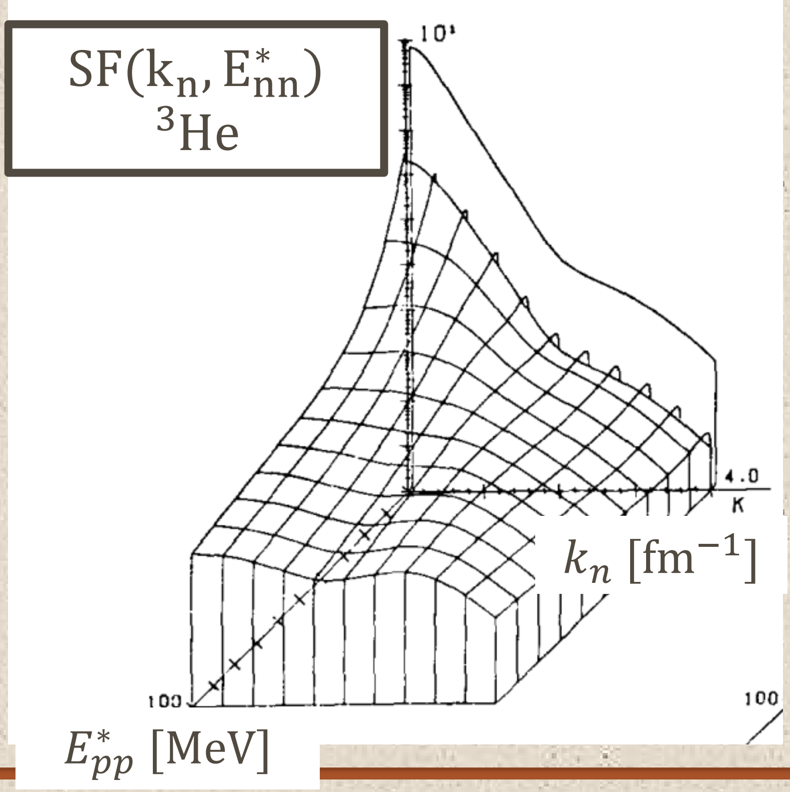
The $nn\Lambda$ is expected to be resonance state ($\Delta s > 0.05$) within the experimental error.

Spectral function of ^{3}He

One of the nucleon momentum in ^{3}H was calculated with spectral function (SF)

However, spectral function of ^{3}H could not reproduced

→ Using SF of ^{3}He assuming charge symmetry



The relative momentum was written by the excited energy of residual system

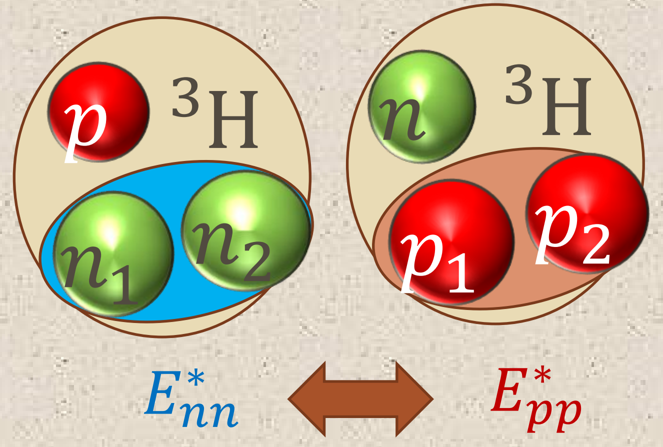
$$E^*_{pp} = \frac{|\vec{p}_{rel}|^2}{2\mu} = \frac{|\vec{p}_{rel}|^2}{m_p}$$

The proton momentum in ^{3}He

$$|\vec{p}_p^{\,3\mathrm{He}}| = \sqrt{m_p E^*_{pp} + \frac{|\vec{p}_n|^2}{4} \mp |\vec{p}_n|(m_p E^*_{pp})\cos\theta}$$

$$|\vec{p}_n^{\,3\mathrm{H}}| = \sqrt{m_n E^*_{pp} + \frac{|\vec{p}_p|^2}{4} \mp |\vec{p}_p|(m_n E^*_{pp})\cos\theta}$$

Mirror system



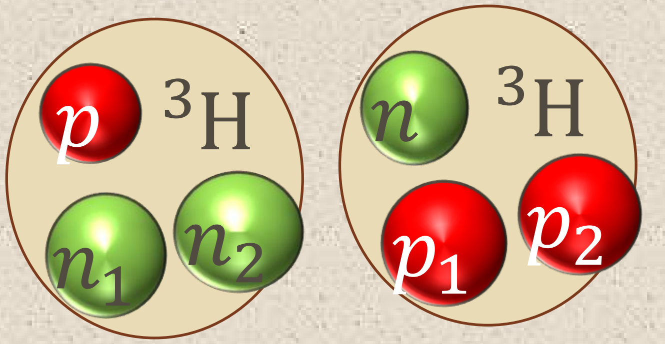
Assuming charge symmetry

$$E^*_{nn} = E^*_{pp}$$

Charge symmetry between ^{3}H and ^{3}He

One of the neutron (proton) momenta in $^3\mathrm{H}$ ($^3\mathrm{He}$) were calculated by excited energy of residual systems (E^*_{pp})

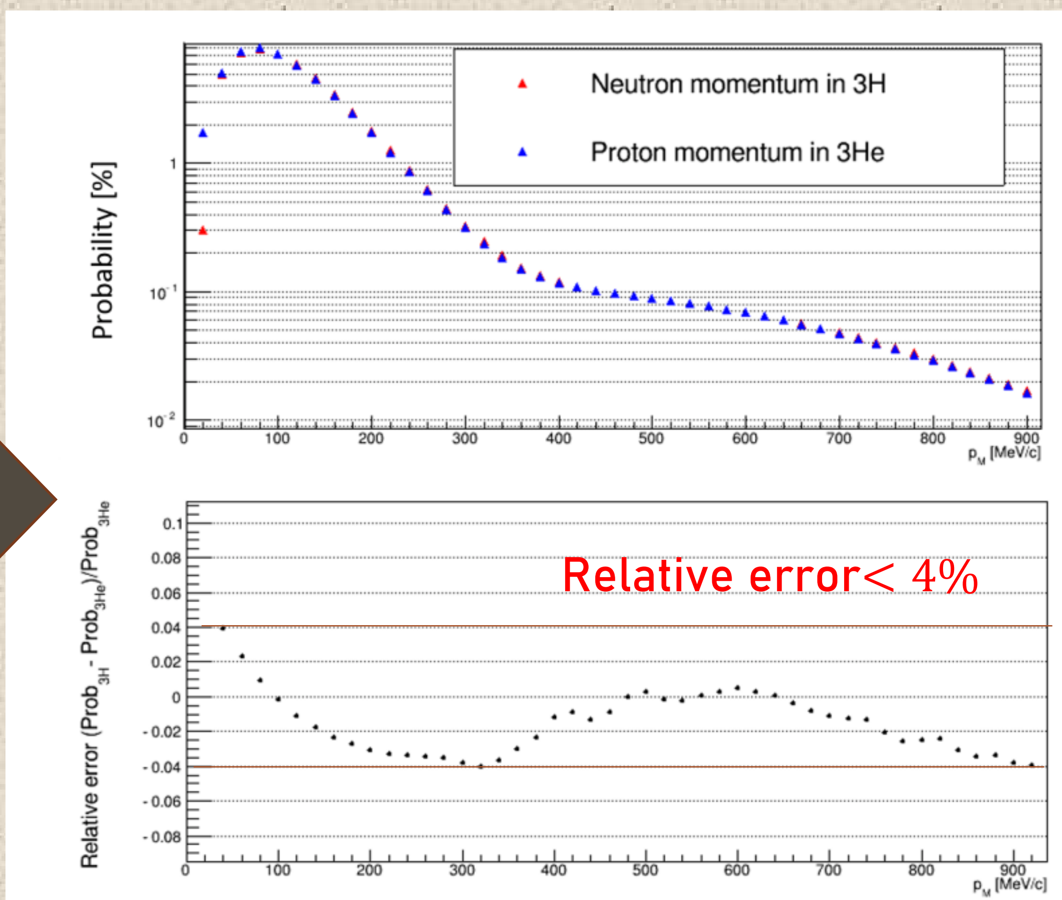
The relative error of the nucleon momentum is expected to be same as one of the Fermi momentum



$$\mathrm{R_{resi}} = \frac{\mathrm{P_{3H}}\left(\vec{p}^{\,\mathrm{nn}}_{\mathrm{n_1}}\right) - \mathrm{P_{3He}}\left(\vec{p}^{\,\mathrm{pp}}_{\mathrm{p_1}}\right)}{\mathrm{P_{3He}}\left(\vec{p}^{\,\mathrm{pp}}_{\mathrm{p_1}}\right)}$$

$$\mathrm{R_{Fer}} = \frac{\mathrm{P_{3H}}\left(\vec{p}^{\,\mathrm{Fer}}_{\mathrm{n_1}}\right) - \mathrm{P_{3He}}\left(\vec{p}^{\,\mathrm{Fer}}_{\mathrm{p_1}}\right)}{\mathrm{P_{3He}}\left(\vec{p}^{\,\mathrm{Fer}}_{\mathrm{p_1}}\right)}$$

$p^{3\mathrm{H}}_{n_1}$ ↔ $p^{3\mathrm{He}}_{p_1}$



The relative error of the Fermi momentum distribution at each momentum is **less than 4%**

Calculation of final state interaction

There are three parameters $(p_{\Lambda n},\ a,\ r)$