Hypernuclear deformation with antisymmetrized molecular dynamics

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Grand challenges of hypernuclear physics

Interaction: "baryon-baryon interaction"

- **2 body interaction between baryons (Y: hyperon, N: nucleon)**
  - hyperon-nucleon (YN)
  - hyperon-hyperon (YY)

Major issues in hypernuclear physics

Structure: "many-body system of nucleons and hyperon"

- **Addition of hyperon as an impurity in (hyper)nuclei**
  - No Pauli exclusion between N and Y
  - YN interaction is different from NN

→ Structure changes
Unique structure, ⋯ etc.

Today: "deformation of hypernuclei"

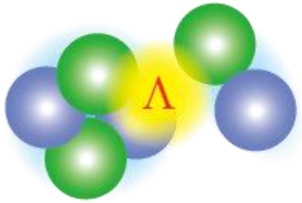
Structure of Λ hypernuclei

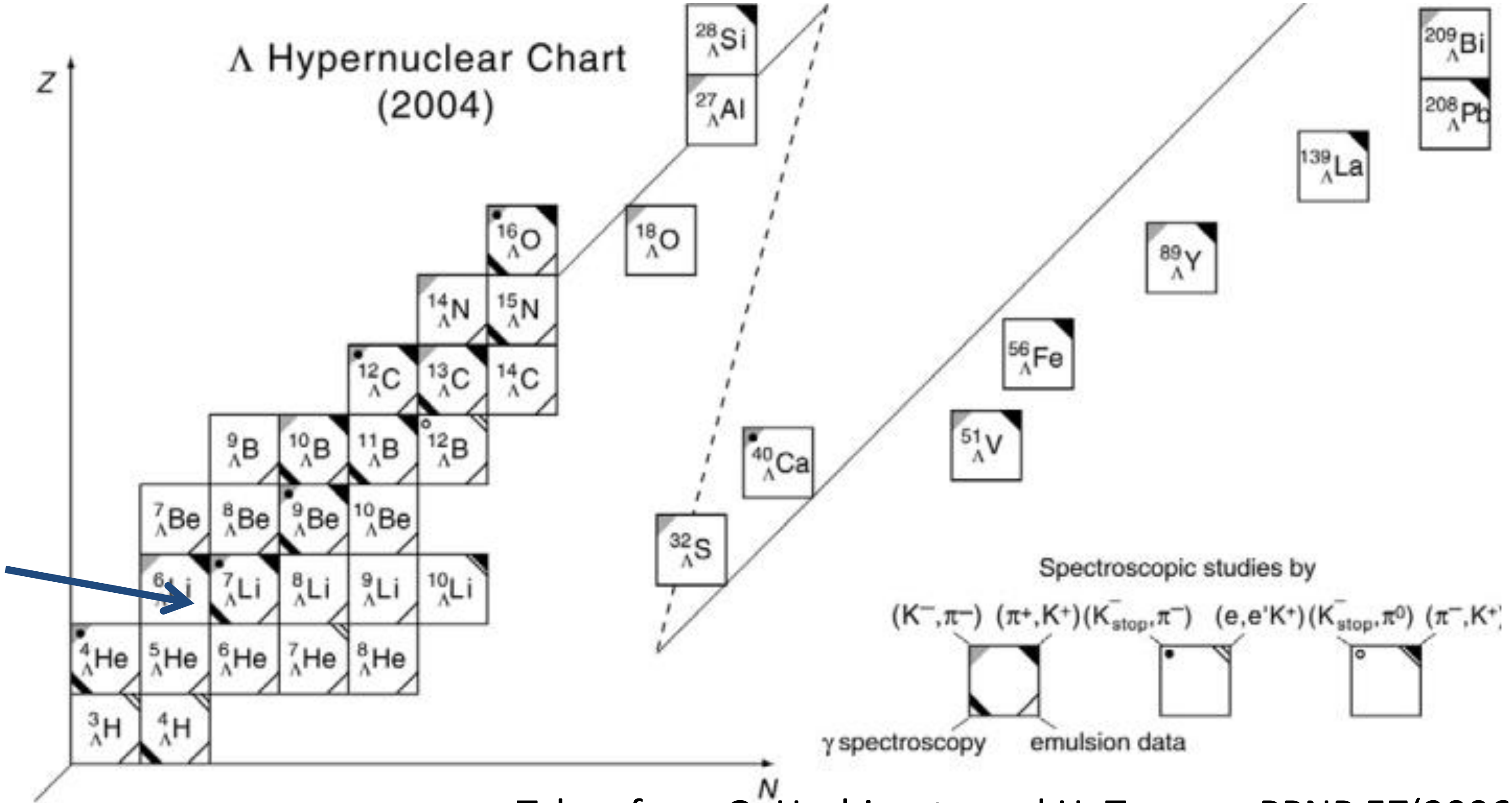
◆Λ hypernuclei observed so far

- **Concentrated in light Λ hypernuclei**
- **Most have well-developed cluster structure**

Light Λ hypernuclei

Developed cluster





Taken from O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

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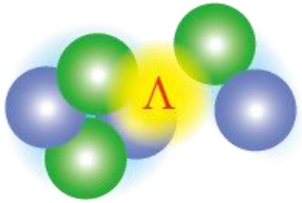
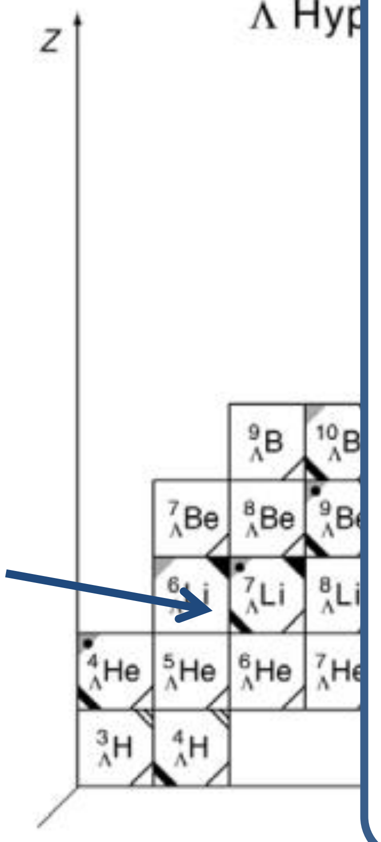




Example of “impurity effect”

Example: $^{7}_{\Lambda}$Li

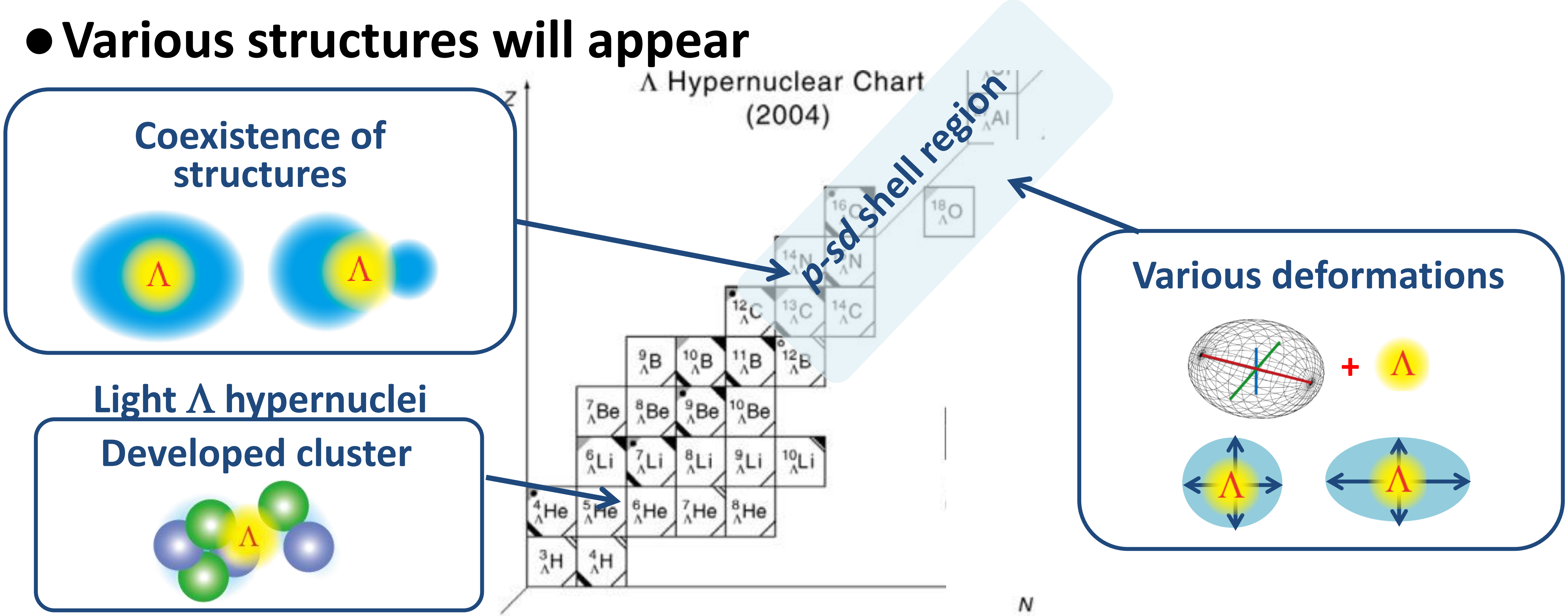
Motoba, *et al.*, PTP**70**,189 (1983)
Hiyama, *et al.*, PRC**59** (1999), 2351.
Tanida, *et al.*, PRL**86** (2001), 1982.

- Λ reduces inter-cluster distance between α + *d* of the core nucleus ^{6}Li
- Confirmed through B(E2) reduction

Toward heavier and exotic Λ hypernuclei

◆ **Experiments at J-PARC, JLab, *etc*.**

- **Heavier(*sd*-shell) & n-rich hypernuclei can be produced**
- **Various structures will appear**



Coexistence of structures

Light Λ hypernuclei

Developed cluster

***p-sd* shell region**

Various deformations

Today: “deformation of hypernuclei”

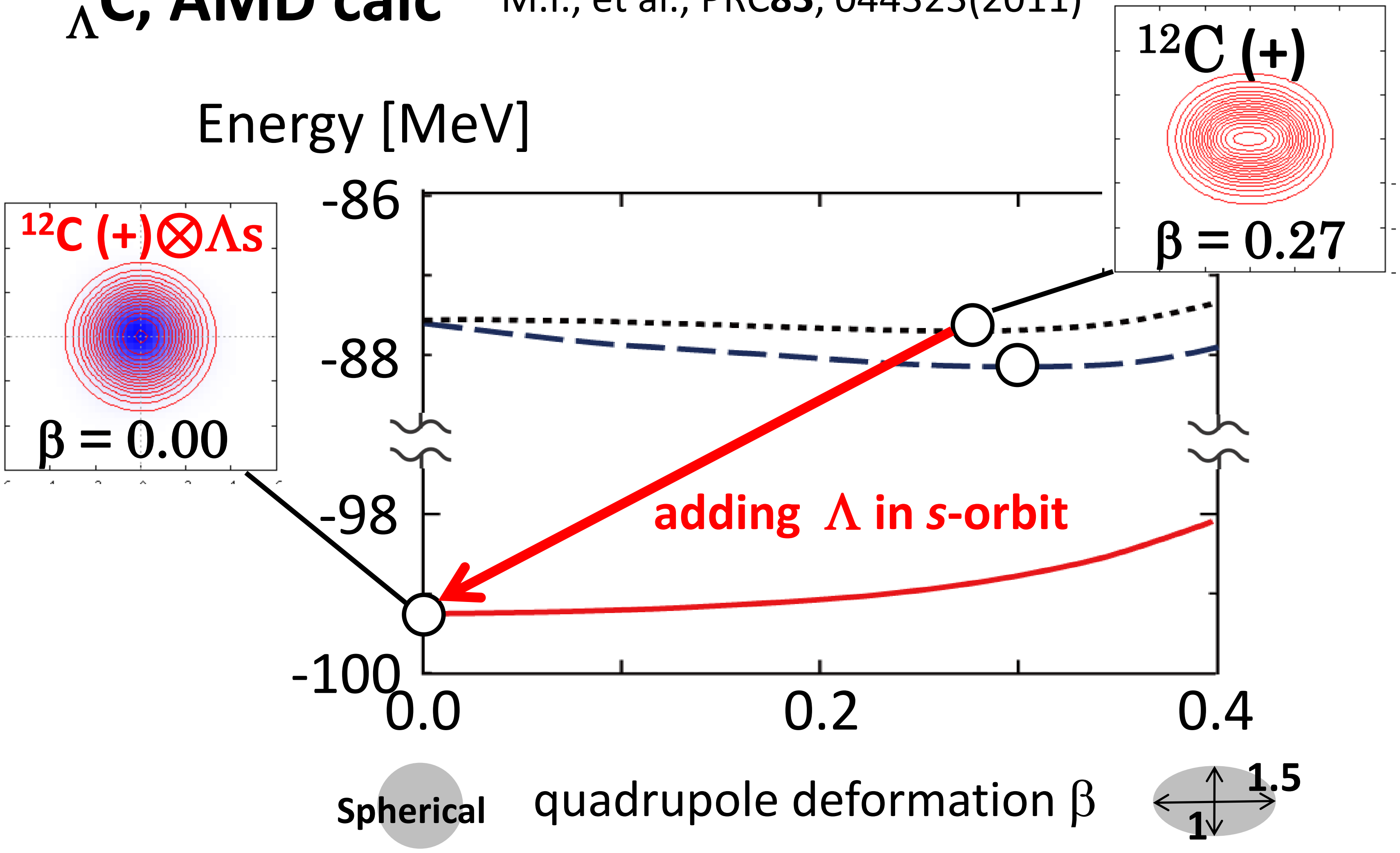
What is expected in deformed Λ hypernuclei

- **Deformation change**
  - Λ particle can change nuclear deformation
- **Difference of B_Λ depending on nuclear deformation**
  - Energy shifts in excitation spectra
- **Coupling of Λ to deformed nuclei shows unique structure**
  - For example, rotational band, mixing of configuration, … etc.

Deformation change by Λ particle

◆Λ particle in s orbit reduces nuclear deformation

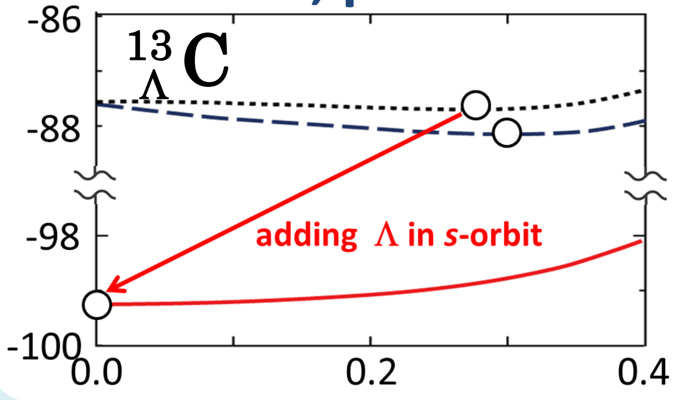
Example: $^{13}{}_{\Lambda}$C, AMD calc M.I., et al., PRC**83**, 044323(2011)



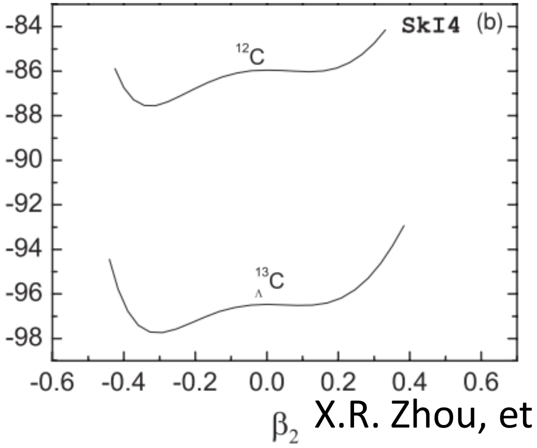
Deformation change by Λ particle

Many authors predict the deformation change by Λ in *s*-orbit

AMD, present

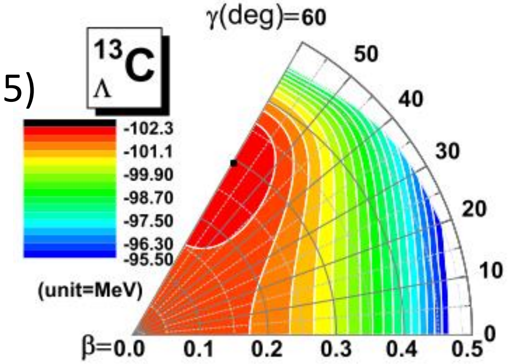


Skyrme-Hartree-Fock (SHF)



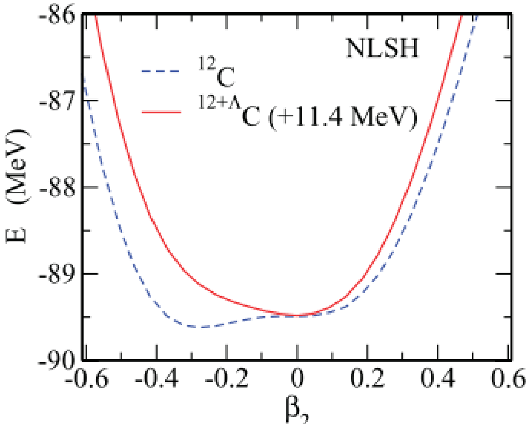
X.R. Zhou, et al., PRC**76**, 034312('07)

J.W. Cui, et al,
PRC**91**,054306('15)

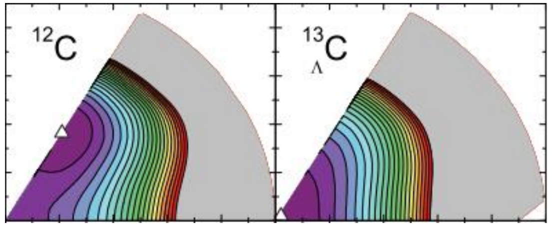


Relativistic mean-field (RMF)

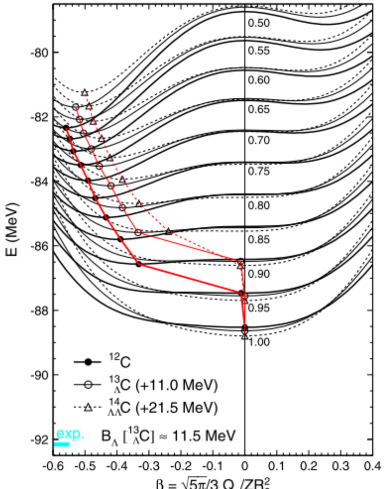
Win and Hagino, PR C**78**, 054311('08)



B.N. Lu, et al., PRC**84**, 014328 ('11)



RMF & SHF



H. J. Schulze, et al.,
PTP**123**, 569('10)

Deformations/level structure with beyond-mean-field

J.W. Cui, X.R. Zhou, H.J. Schulze,
PRC**91**,054306('15)

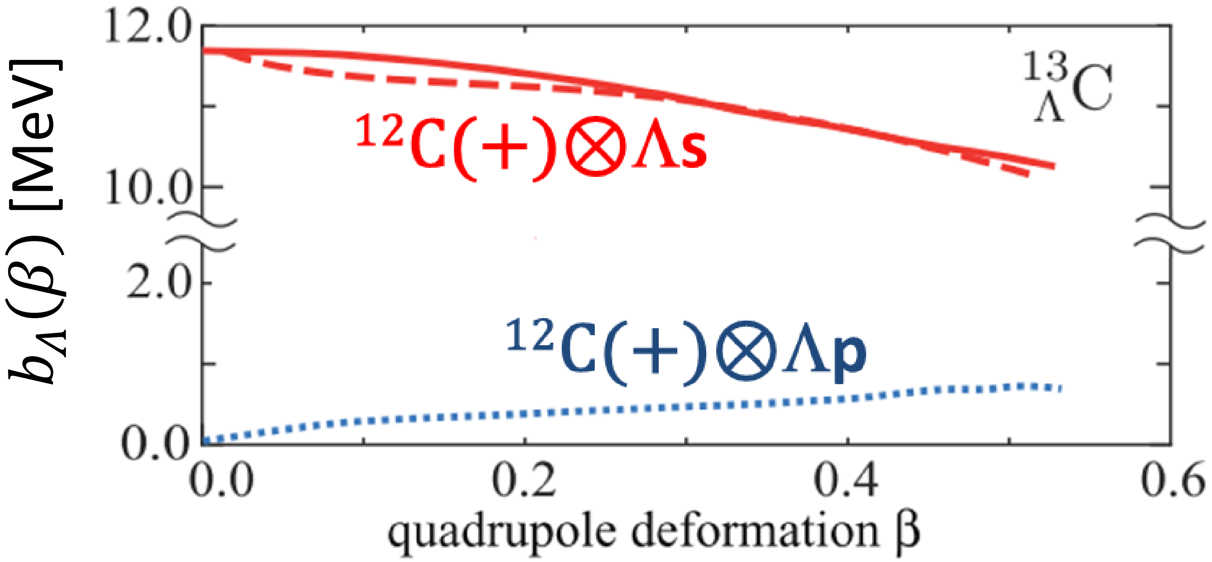
H. Mei, K. Hagino, J.M. Yao, T. Motoba,
PRC91, 064305('15)

etc.

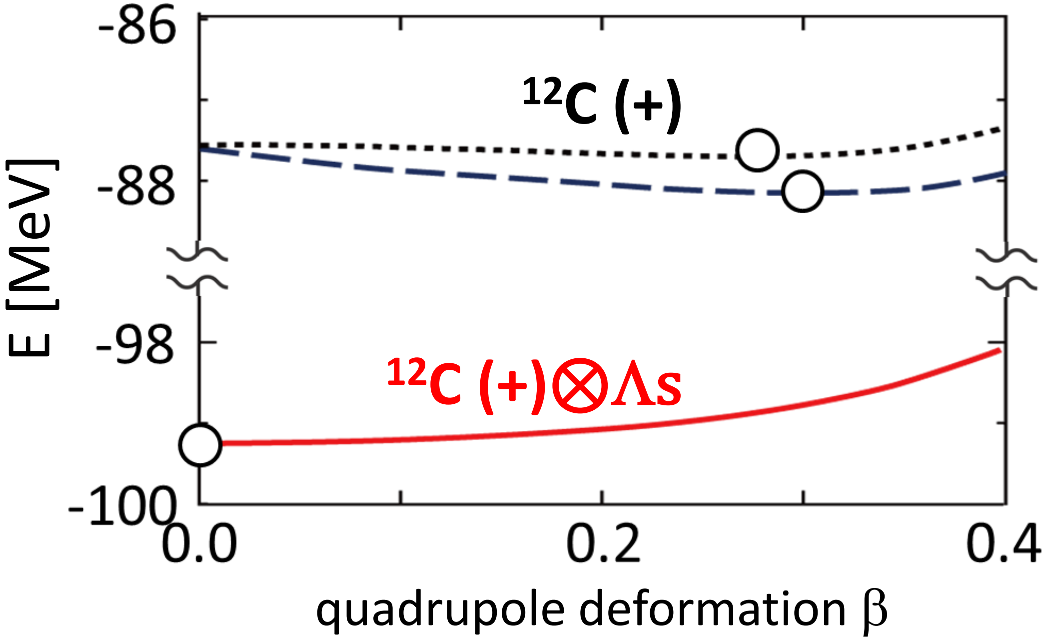
Deformation change by Λ particle

- Λ in s orbit is deeply bound with smaller deformation corresponding to larger overlap between Λ and N
- Deformation change is caused by competition between Λ binding energy and nuclear energy surface

$$b_\Lambda(\beta) = E_{core}(\beta) - E_\Lambda(\beta)$$



cf. energy curve



What is expected in deformed Λ hypernuclei

- **Deformation change**
  - Λ particle can change nuclear deformation

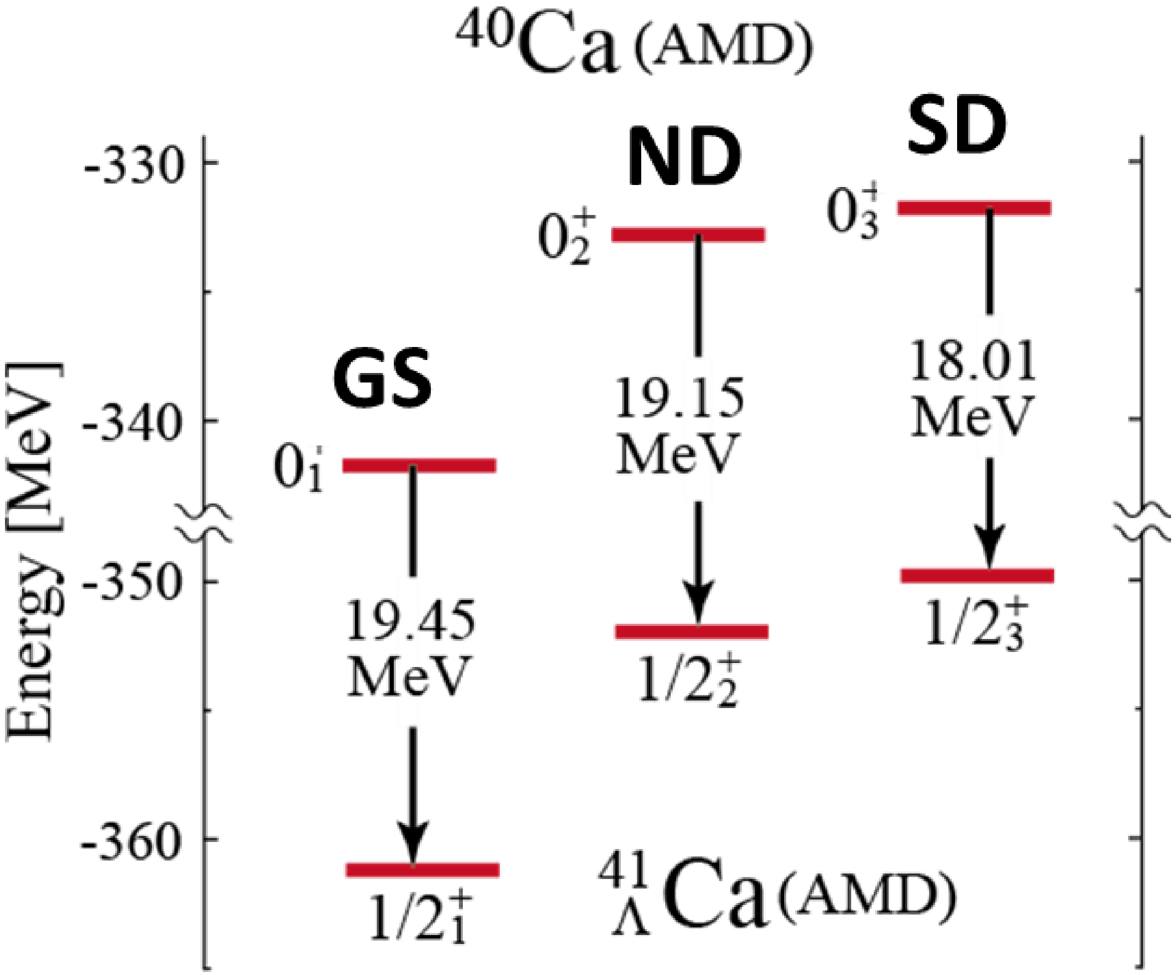
- **Difference of B_{Λ} depending on nuclear deformation**
  - Energy shifts in excitation spectra

- **Coupling of Λ to deformed nuclei shows unique structure**
  - For example, rotational band, mixing of configuration, … etc.

Difference of B_Λ depending on nuclear deformation

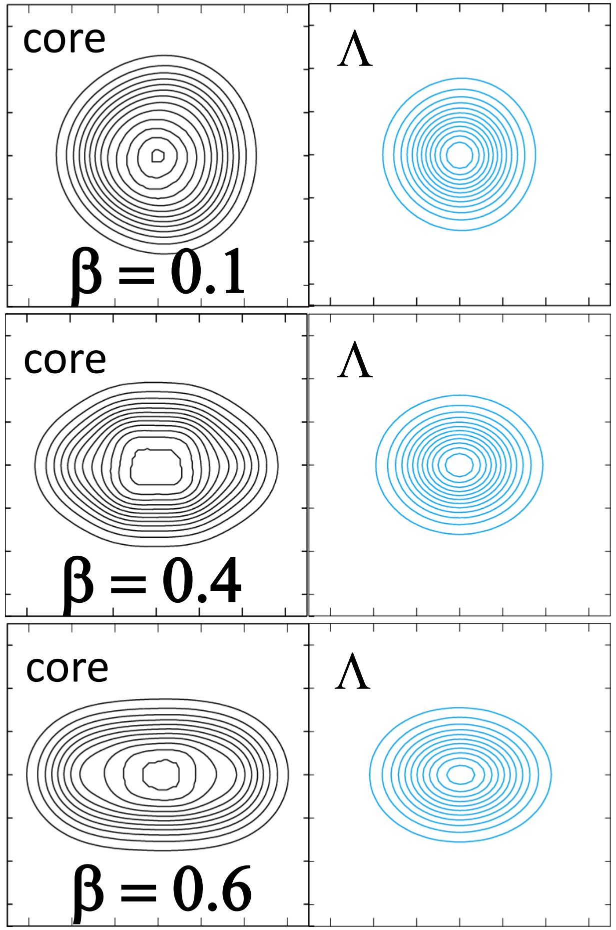
- **B_Λ is sensitive to nuclear deformation through overlap b/w Λ and N**

M. Isaka, *et al.*, PRC**89**, 024310(2014)



Note: Deformation change is quite small

Overlap: $I = \int d^3r \rho_N(\boldsymbol{r})\rho_\Lambda(\boldsymbol{r})$ [fm^{-3}]



GS
$I = 0.1364$

ND
$I = 0.1356$

SD
$I = 0.1336$

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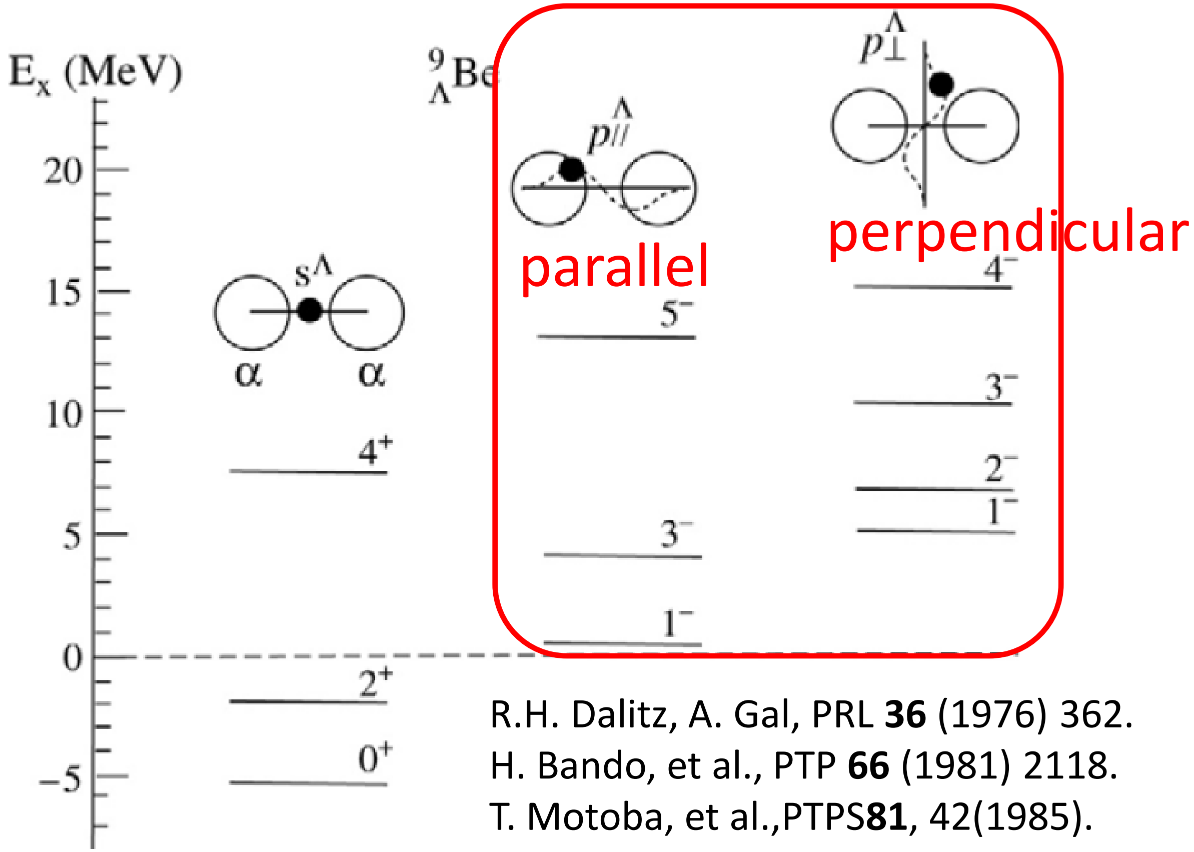
Today {
- triaxial deformation of Mg nuclei: ${}^{27}_{\Lambda}$Mg, future exp at JLab
- Rotational bands by Λ in p orbit coupled to the core

Coupling of Λ in *p*-orbit: *p*-states of $^{9}_{\Lambda}$Be

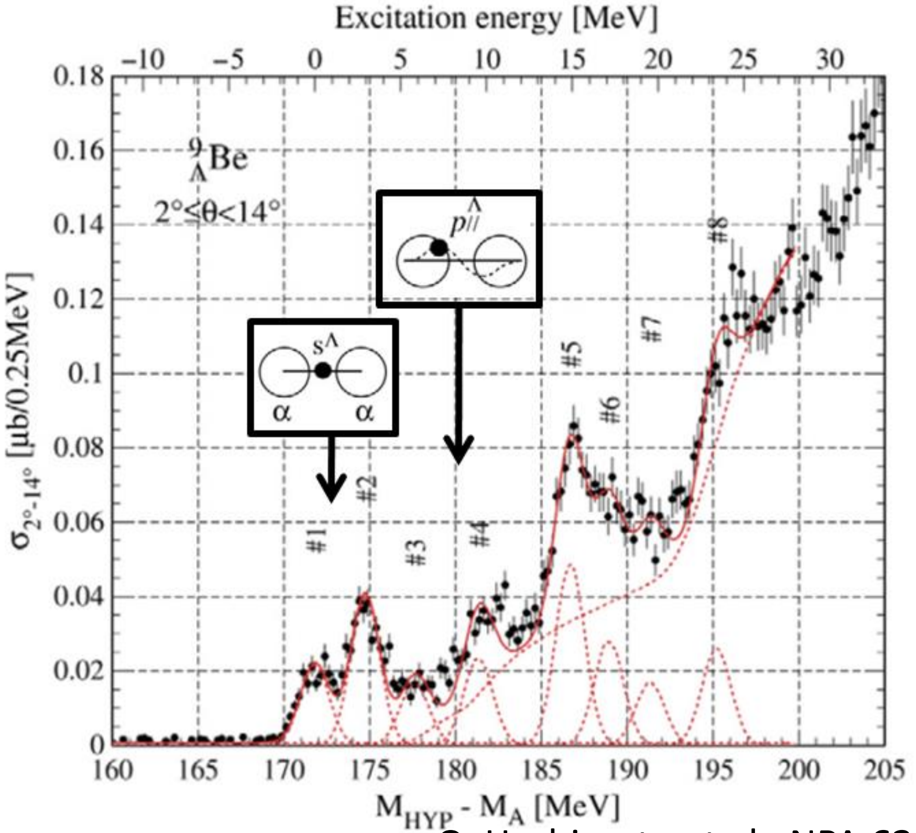
$^{9}_{\Lambda}$Be: axially symmetric 2α clustering

Two rotational bands as *p*-states
- **Anisotropic *p* orbit of Λ hyperon**
- **Axial symmetry of 2α clustering**

→ *p*-orbit parallel to/perpendicular to the 2α clustering



R.H. Dalitz, A. Gal, PRL **36** (1976) 362.
H. Bando, et al., PTP **66** (1981) 2118.
T. Motoba, et al.,PTPS**81**, 42(1985).



O. Hashimoto et al., NPA **639** (1998) 93c.

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Theoretical Framework: HyperAMD

M.I., *et al.*, PRC**83**(2011) 044323
M. I., *et al.*, PRC**83**(2011) 054304

HyperAMD: Antisymmetrized Molecular Dynamics for hypernuclei

◆Hamiltonian

$$\hat{H} = \hat{T}_N + \hat{V}_{NN} + \hat{T}_\Lambda + \hat{V}_{\Lambda N} - \hat{T}_g$$

NN : Gogny D1S, Volkov No.2
ΛN : YNG interaction (ESC14)

◆Wave function

- **Nucleon part : Slater determinant**
  Spatial part of s.-p. w.f. is described as Gaussian packets

$$\varphi_N(\vec{r}) = \frac{1}{\sqrt{A!}} \det[\varphi_i(\vec{r}_j)]$$

$$\varphi_i(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \nu_\sigma (r - Z_i)_\sigma^2\right] \chi_i \eta_i$$

$$\chi_i = \alpha_i \chi_\uparrow + \beta_i \chi_\downarrow$$

- **Single-particle w.f. of Λ hyperon:**
  Superposition of Gaussian packets

$$\varphi_\Lambda(r) = \sum_m c_m \varphi_m(r)$$

$$\varphi_m(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \mu\nu_\sigma (r - z_m)_\sigma^2\right] \chi_m$$

$$\chi_m = a_m \chi_\uparrow + b_m \chi_\downarrow$$

- **Total w.f. :**

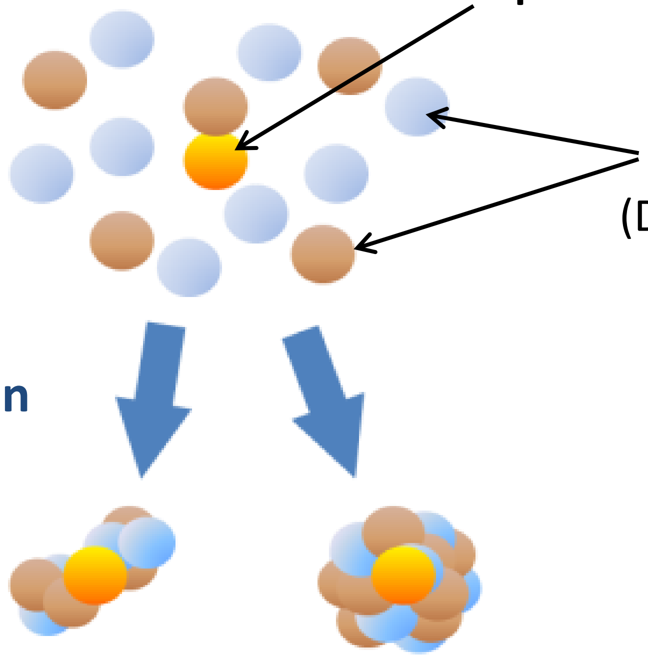
$$\psi(\vec{r}) = \sum_m c_m \varphi_m(r_\Lambda) \otimes \frac{1}{\sqrt{A!}} \det[\varphi_i(\vec{r}_j)]$$

Theoretical framework: HyperAMD

◆Procedure of the calculation

Variation

- Imaginary time development method: $$\frac{dX_i}{dt} = \frac{\kappa}{\hbar}\frac{\partial H^{\pm}}{\partial X_i^{*}} \qquad \kappa < 0$$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$



Initial w.f.

Λ particle

nucleons

(Described by Gaussian wave packets)

Energy variation

cluster

mean-field

Theoretical framework: HyperAMD

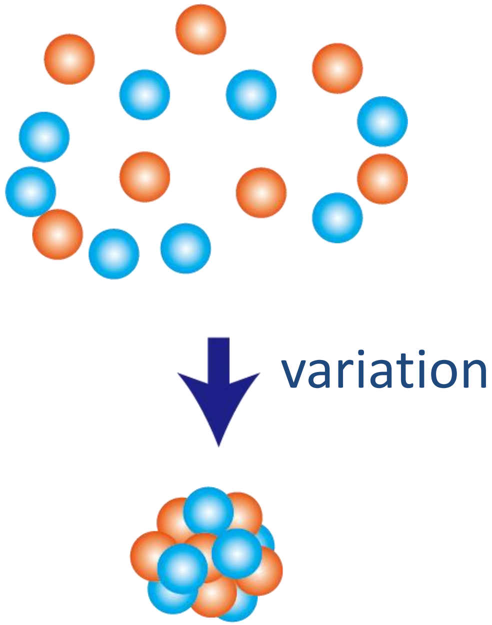
◆Procedure of the calculation

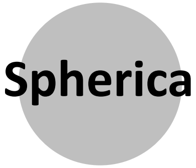
- **Energy variation with constraint on nuclear quadrupole deformation**

Described by (β,γ)

e.g.) ^{12}C

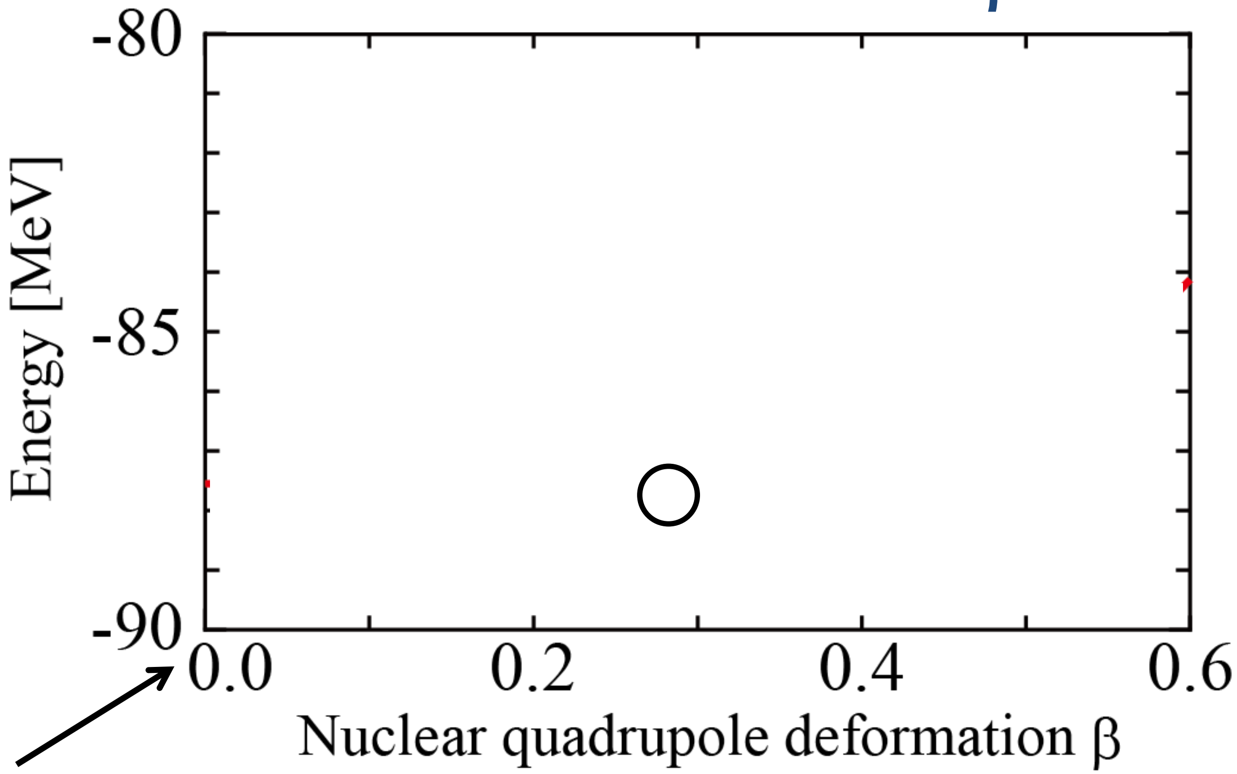
Initial w.f.





$\beta = 0.00$

without constraint on β



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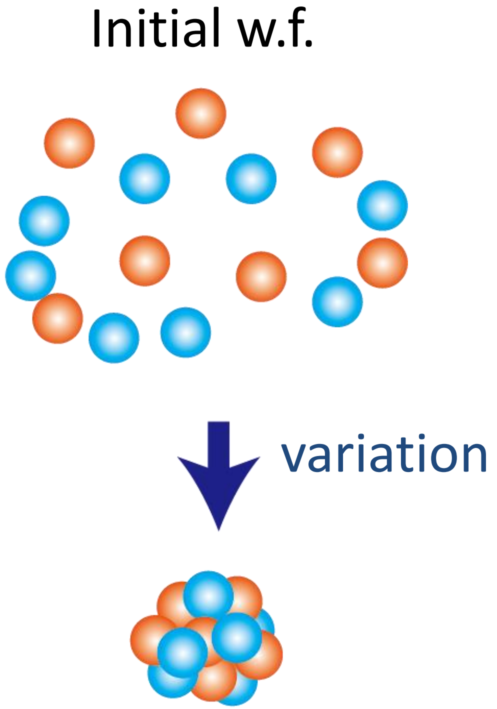
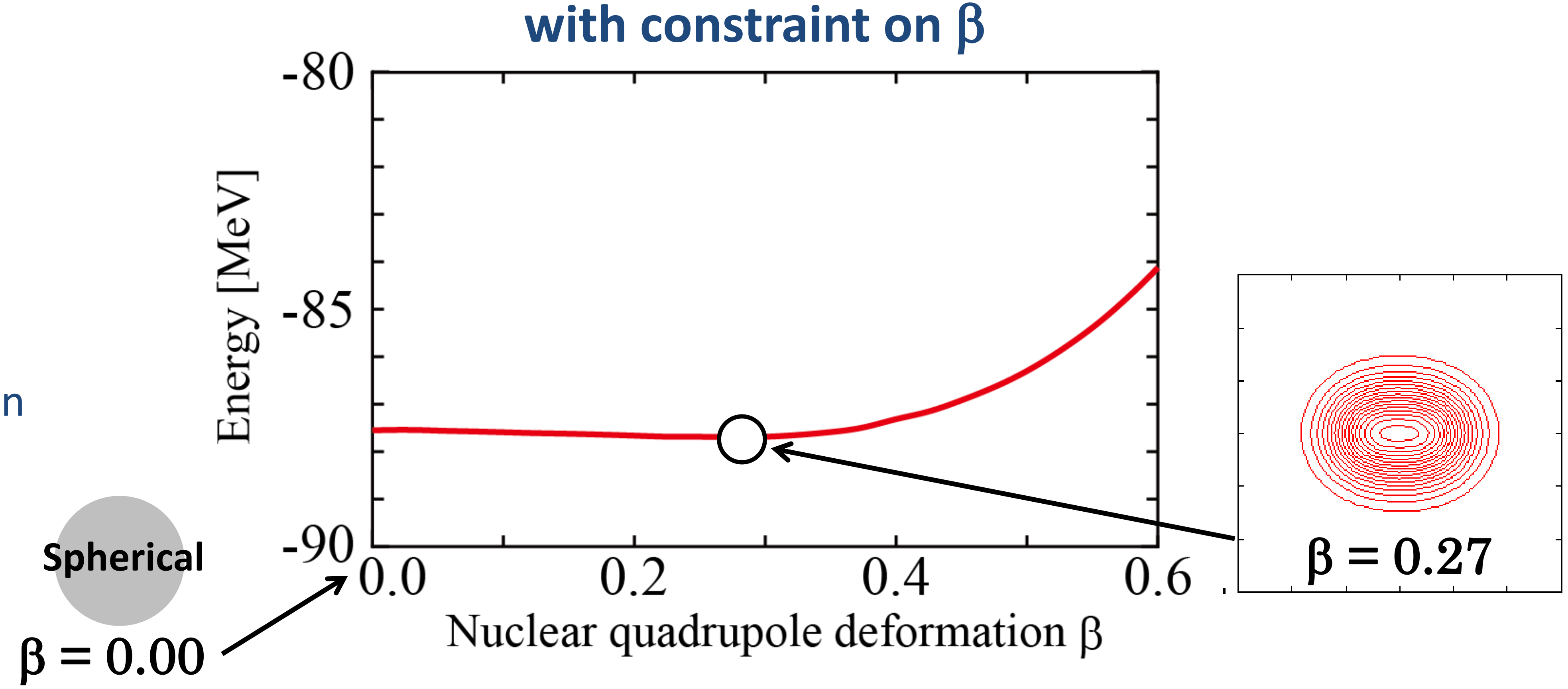
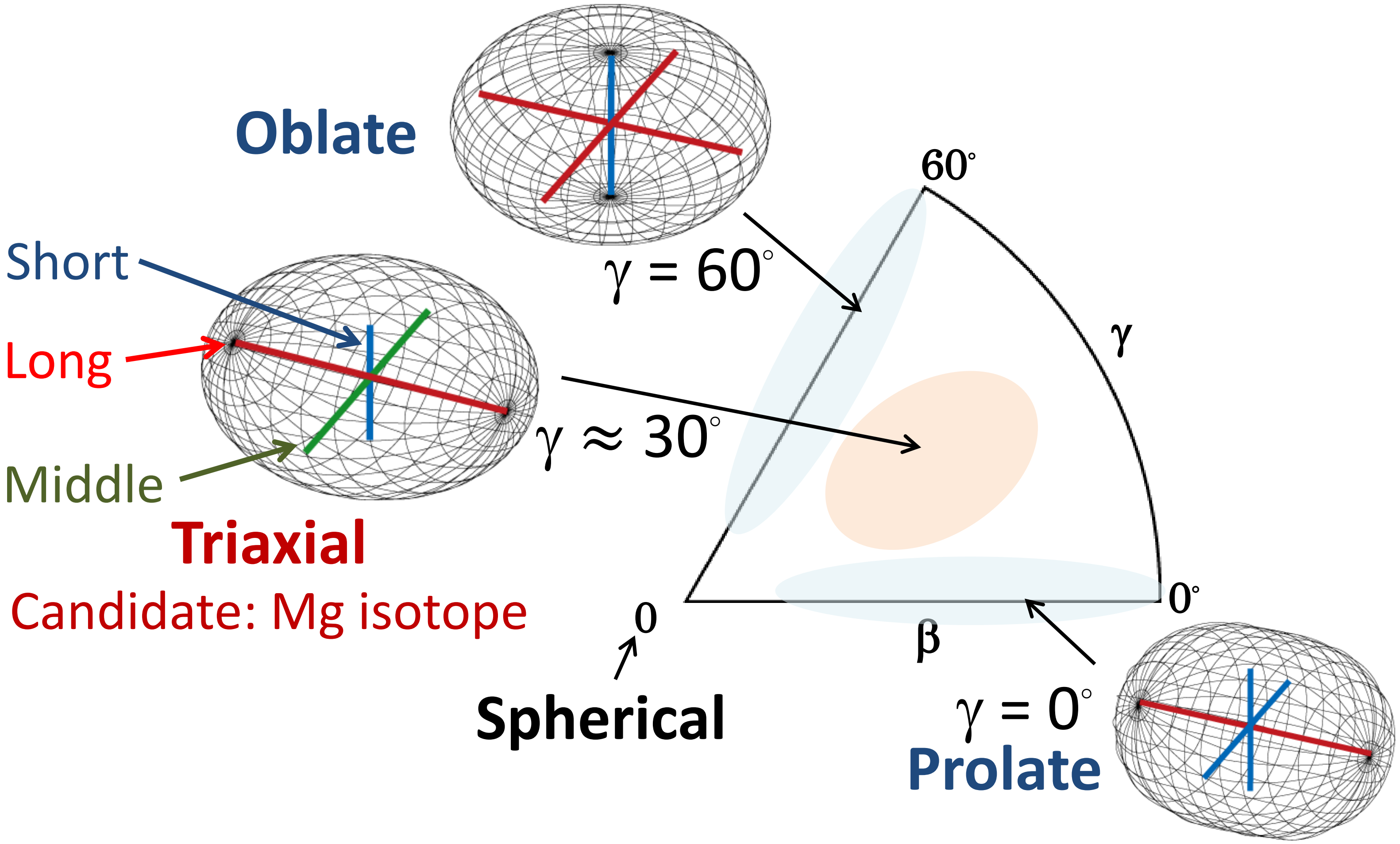
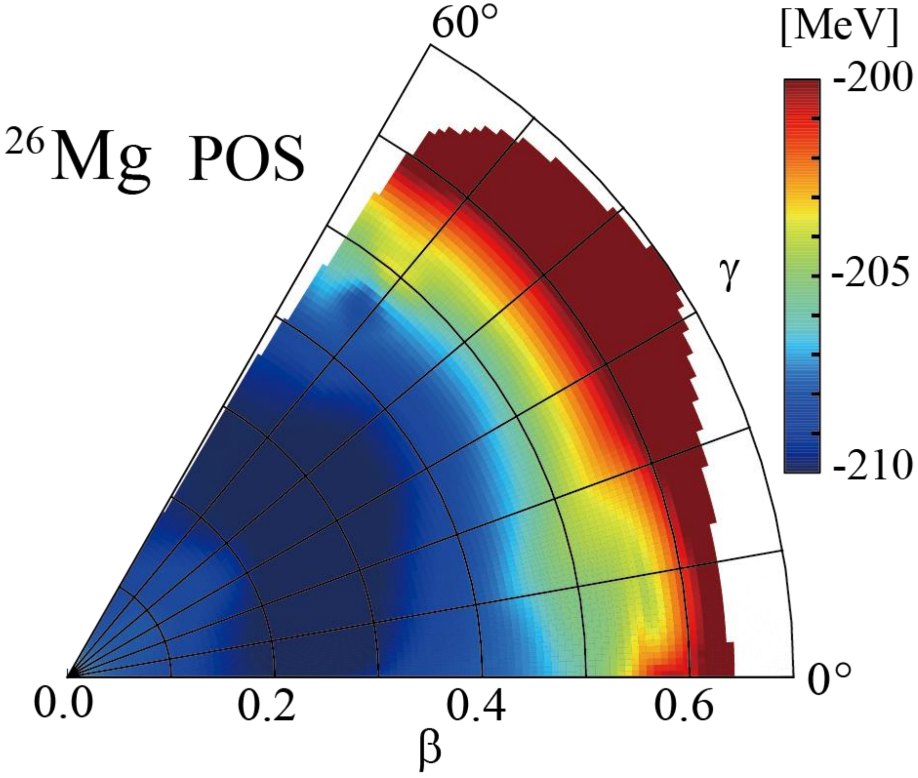




Quadrupole deformation (β, γ)





- Energy variation is performed at each (β, γ)
- *p* states are obtained by constraint on Λ single particle wf: $V_f = \lambda \sum_f |\varphi_f\rangle\langle\varphi_f|$

Theoretical Framework: HyperAMD

M.I., *et al.*, PRC**83**(2011) 044323
M. I., *et al.*, PRC**83**(2011) 054304

◆Procedure of the numerical calculation

Variation

- Imaginary time development method: $\frac{dX_i}{dt} = \frac{\kappa}{\hbar}\frac{\partial H^{\pm}}{\partial X_i^{*}}$ $\quad \kappa < 0$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$

Angular Momentum Projection

$$\left|\Phi^{s}_{K}; JM\right\rangle = \int d\Omega D^{J*}_{MK}(\Omega) R(\Omega) \left|\Phi^{s+}\right\rangle$$

Generator Coordinate Method (GCM)

- Superposition of intrinsic wave functions with different configuration
- Diagonalization of $H^{J\pm}_{sK,s'K'}$ and $N^{J\pm}_{sK,s'K'}$

$$H^{J\pm}_{sK,s'K'} = \left\langle \Phi^{s}_{K}; J^{\pm}M \middle| \hat{H} \middle| \Phi^{s'}_{K'}; J^{\pm}M \right\rangle$$

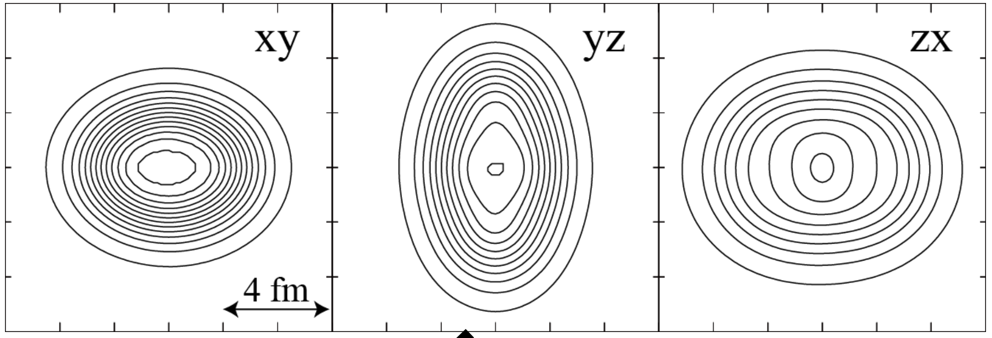
$$N^{J\pm}_{sK,s'K'} = \left\langle \Phi^{s}_{K}; J^{\pm}M \middle| \Phi^{s'}_{K'}; J^{\pm}M \right\rangle$$

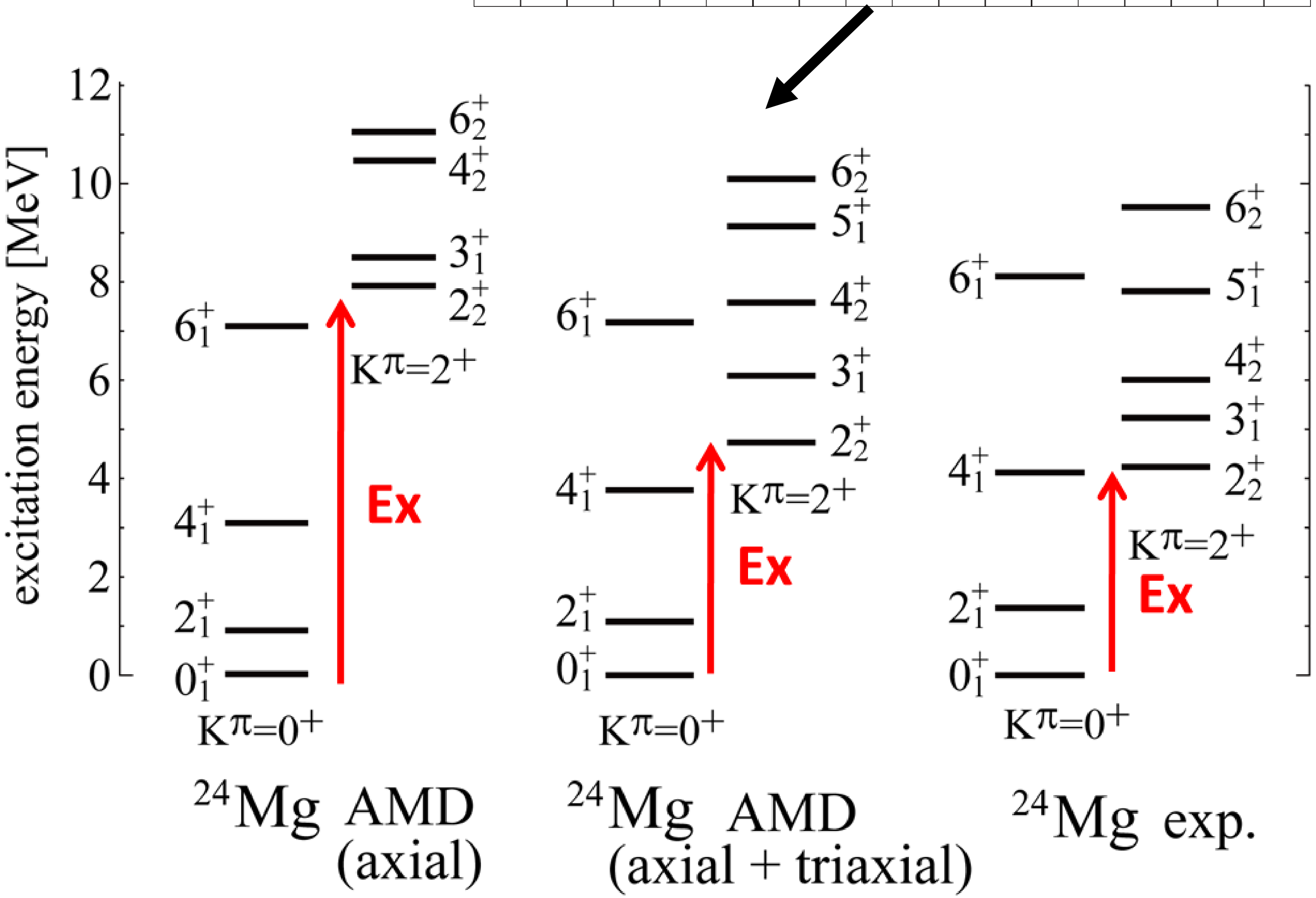
$$\left|\Psi^{J\pm M}\right\rangle = \sum_{sK} g_{sK} \left|\Phi^{s}_{K}; J^{\pm}M\right\rangle$$

Deformation of Mg nuclei

Ex.) ^{24}Mg

- Largely deformed nuclei far from magic number
- Low-lying 2nd 2^+ band indicates triaxial deformation



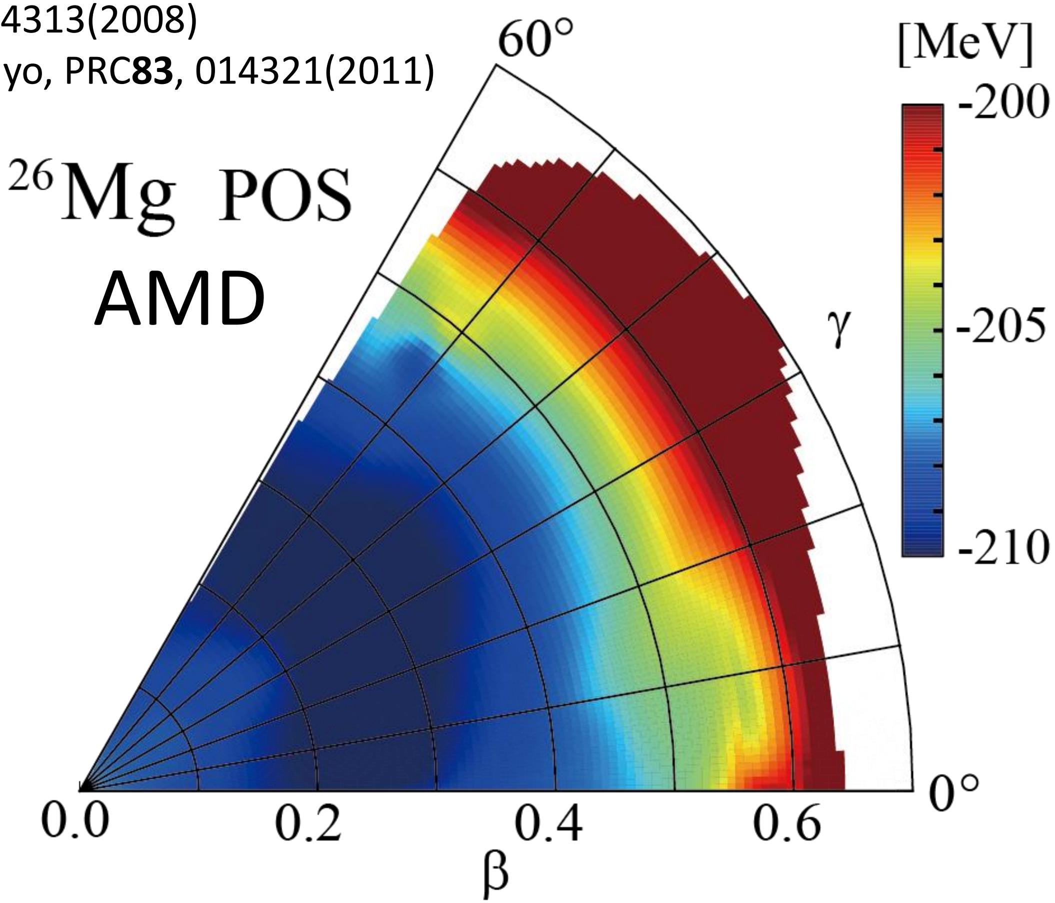


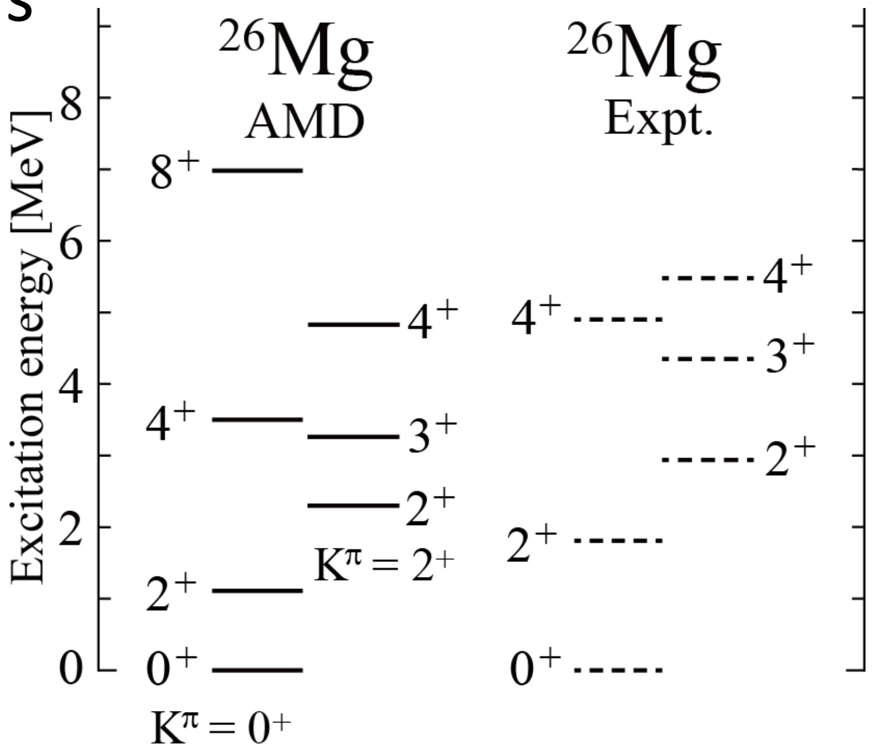
Deformation of Mg nuclei

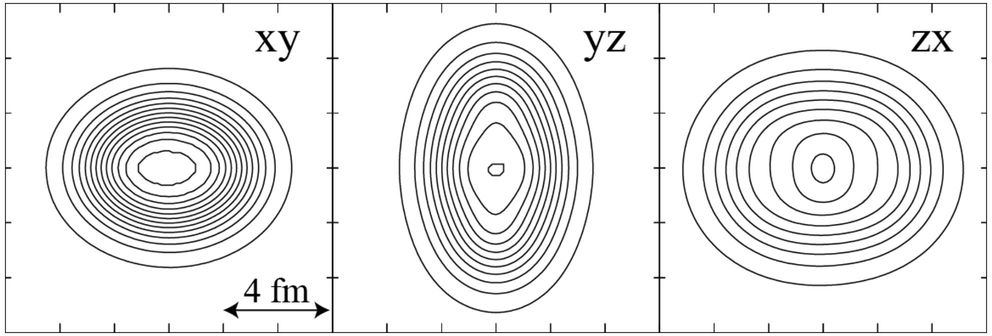
^{26}Mg

- Shell gap in Nilsson diagram: Z=12 (prolate) vs. N=14 (oblate) → triaxial
- β,γ-soft nature is discussed by several authors

Terasaki et al., NPA**621**, 706(1997)
Rodrıguez-Guzman et al., NPA**709**, 201(2002)
Peru et al., PRC**77**, 044313(2008)
Hinohara, Kanada-En'yo, PRC**83**, 014321(2011)



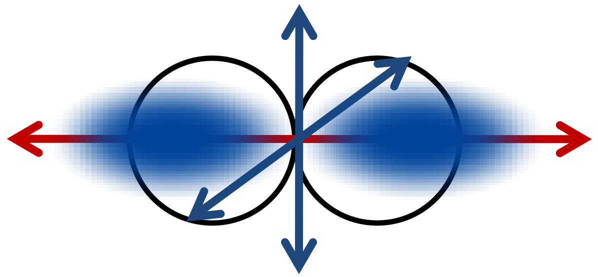




Split of *p*-state in ${}^{9}_{\Lambda}$Be

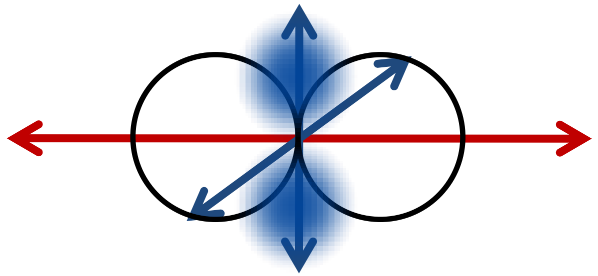
◆${}^{9}_{\Lambda}$Be with 2α cluster structure

<u>*p* orbit parallel to 2α (long axis)</u>

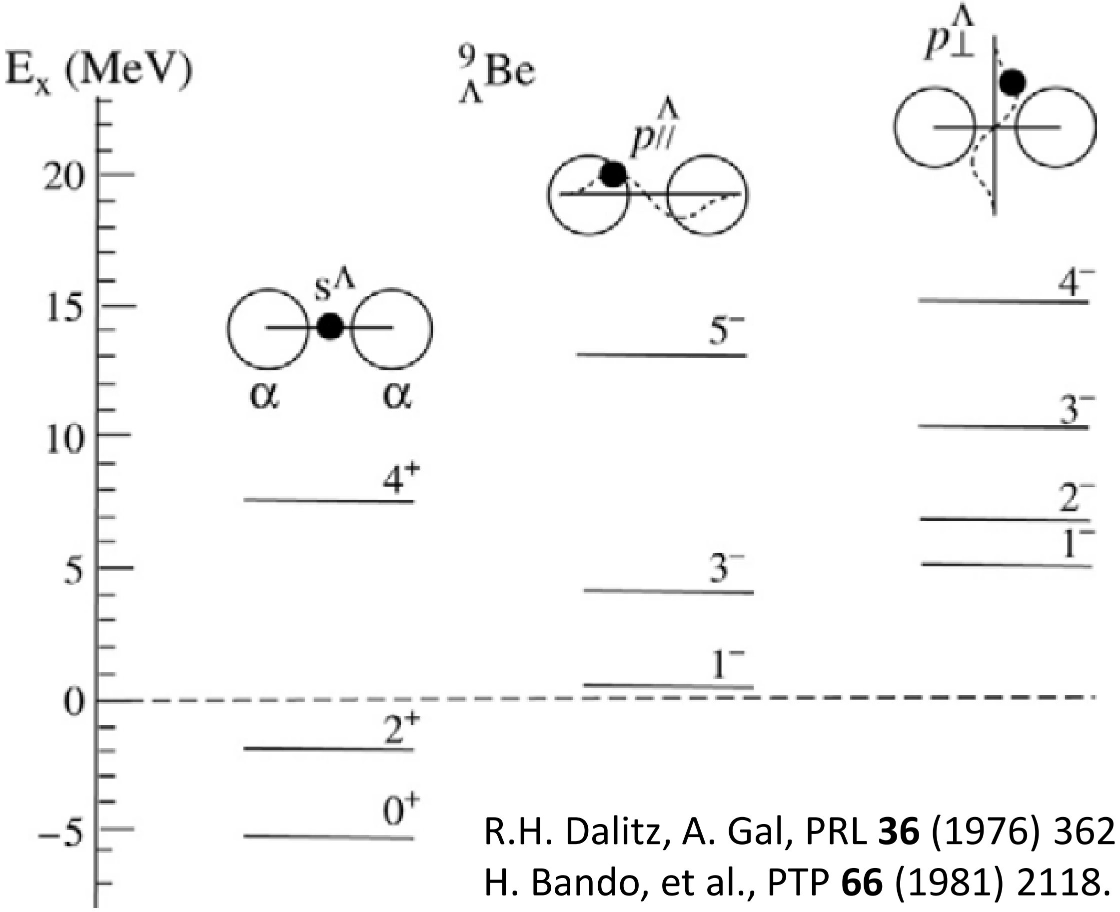


Large overlap
Deeply bound

<u>*p* orbit perpendicular to 2α (short axes)</u>



Small overlap
Shallowly bound



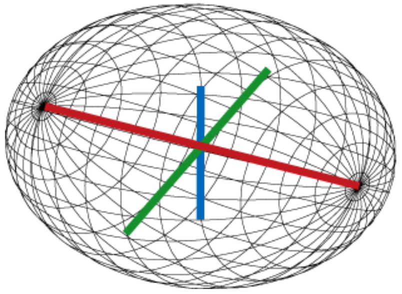
R.H. Dalitz, A. Gal, PRL **36** (1976) 362.
H. Bando, et al., PTP **66** (1981) 2118.
T. Motoba, et al.,PTPS**81**, 42(1985).

***p*-states splits into 2 bands depending on the direction of *p*-orbits**

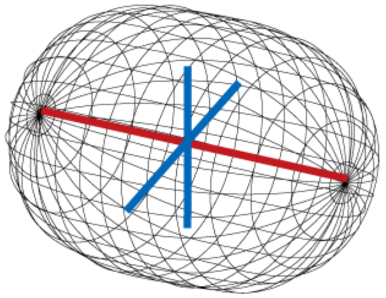
Triaxial deformation

If ^{26}Mg is triaxially deformed nuclei

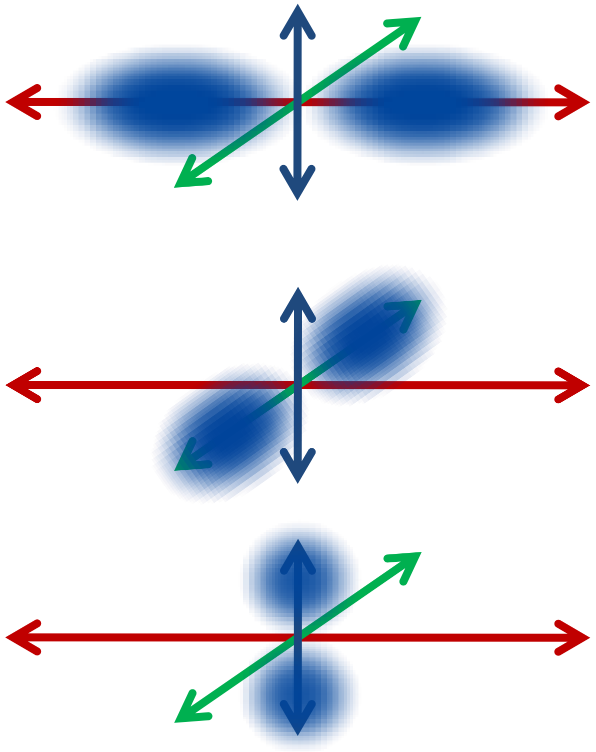
→ *p*-states split into 3 different state



Triaxial deformation



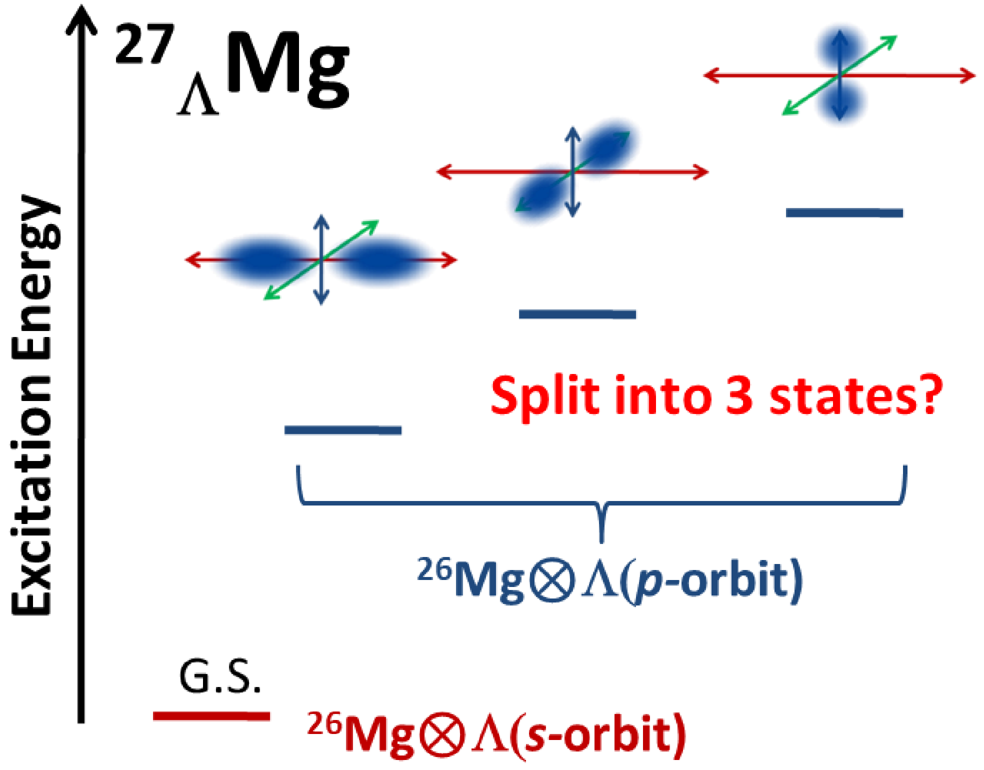
Prolate deformation



Large overlap
Deeply bound

Middle

Small overlap
shallow binding

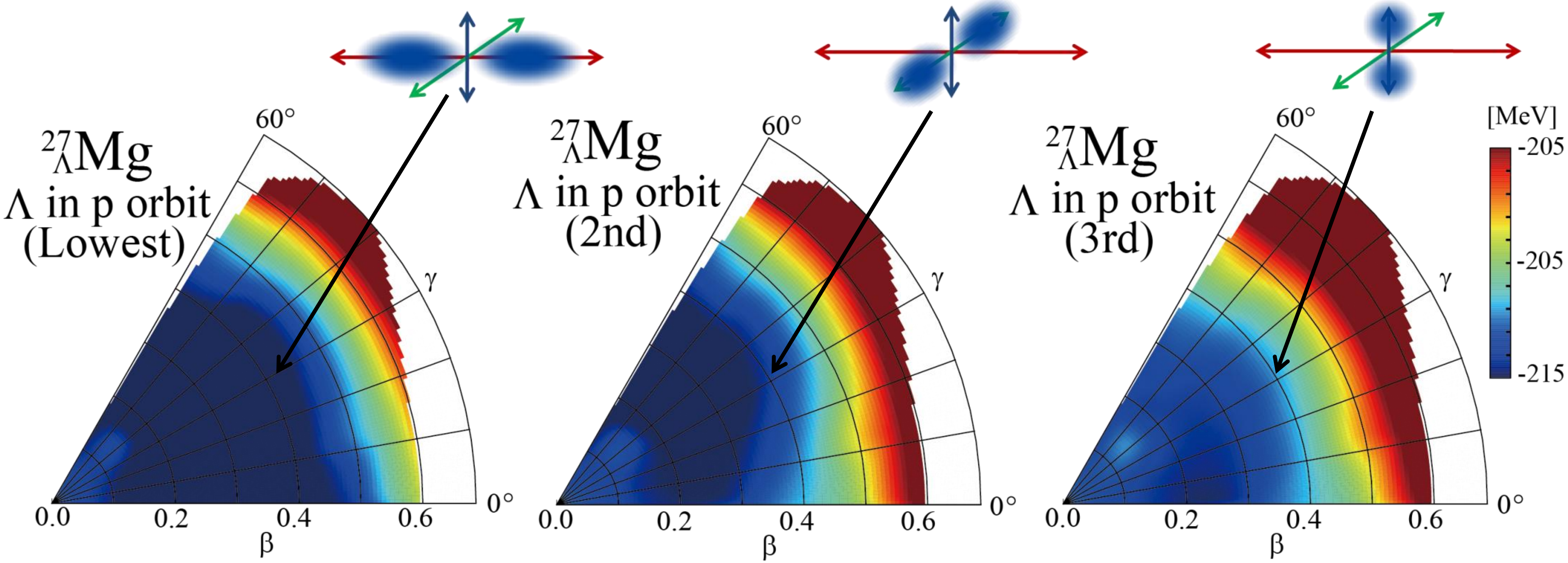


Observing the 3 different *p*-states is strong evidence of triaxial deformation

Energy surface on (β, γ) plane

◆*p*-states of ${}^{27}_{\Lambda}$Mg

- ●**3 different *p* states appear by the energy variation with constraints**
- ●**With different spatial distribution of Λ (in γ ≃ 30 deg. region)**

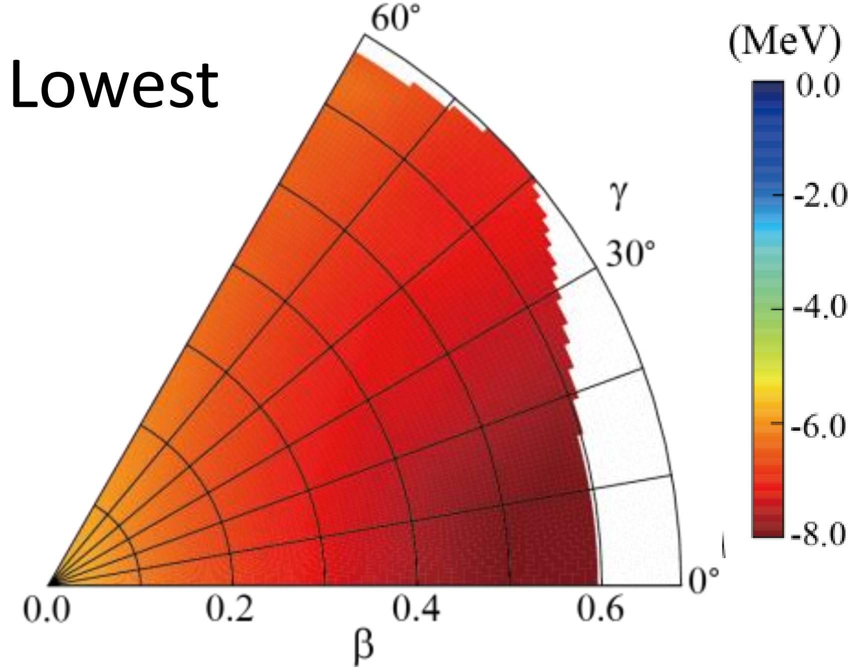


Results： Single particle energy of Λ hyperon

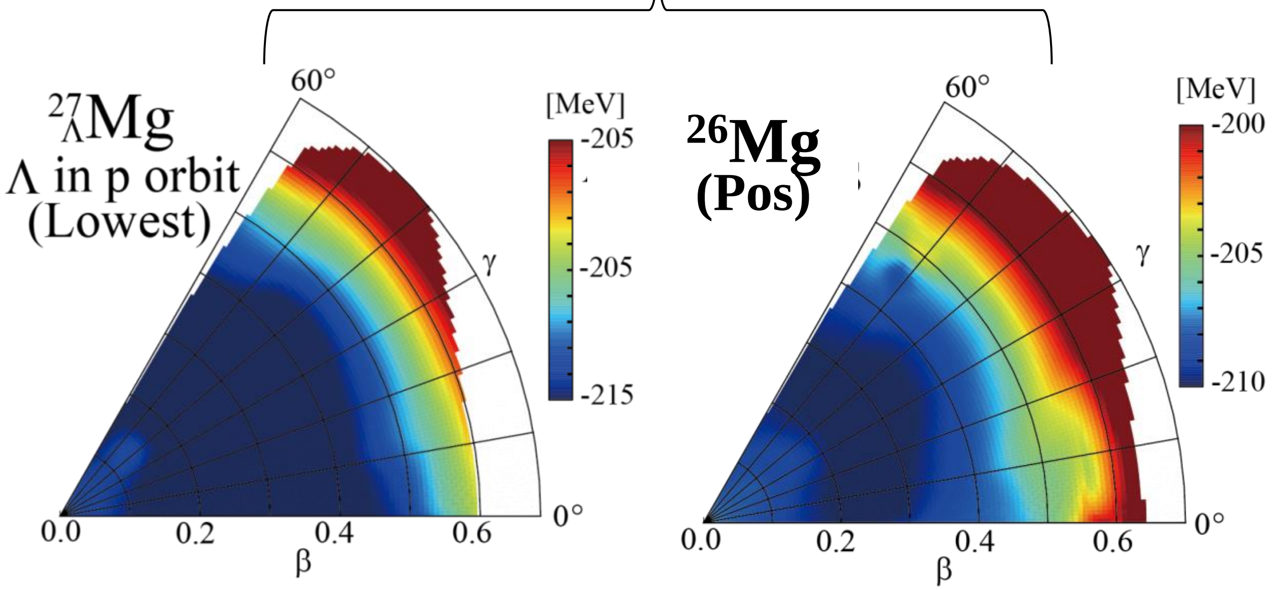
◆Λ single particle energy on (β, γ) plane

$$\varepsilon_{\Lambda}(\beta,\gamma) = E_{\Lambda p}(\beta,\gamma) - E_{core}(\beta,\gamma)$$

$^{27}_{\Lambda}$Mg (AMD, Λ in *p* orbit)



ε$_{\Lambda}$(β,γ): energy difference



Single particle energy of Λ particle is different in each p state corresponding the difference of overlap between Λ and nucleons

Results：Single particle energy of Λ hyperon

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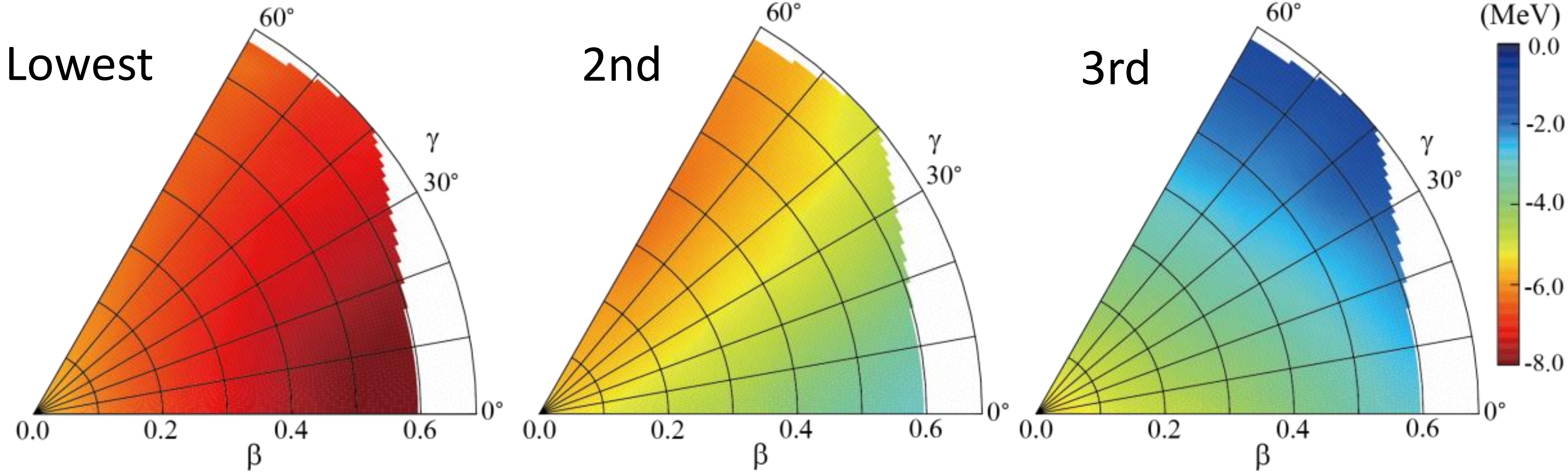
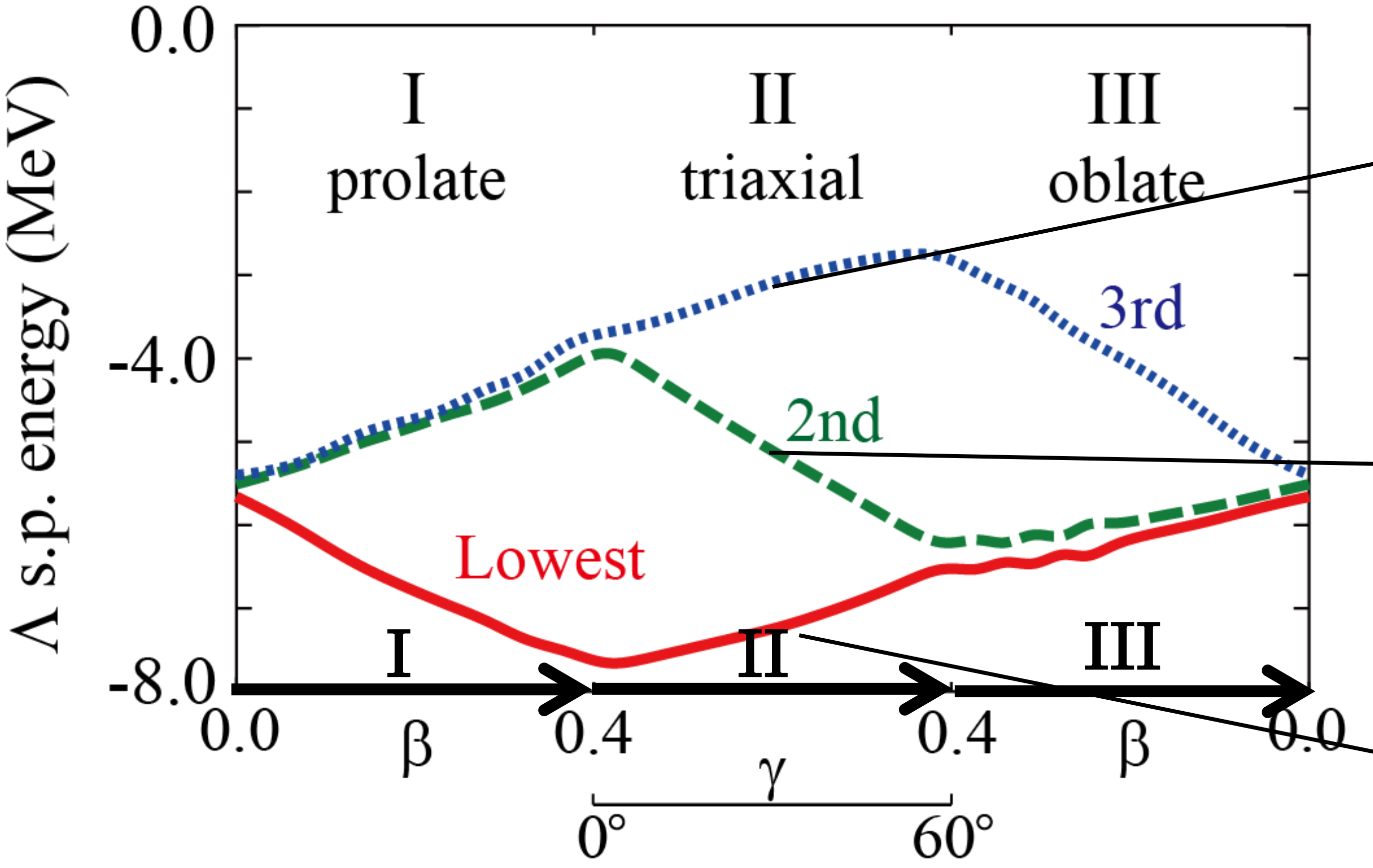


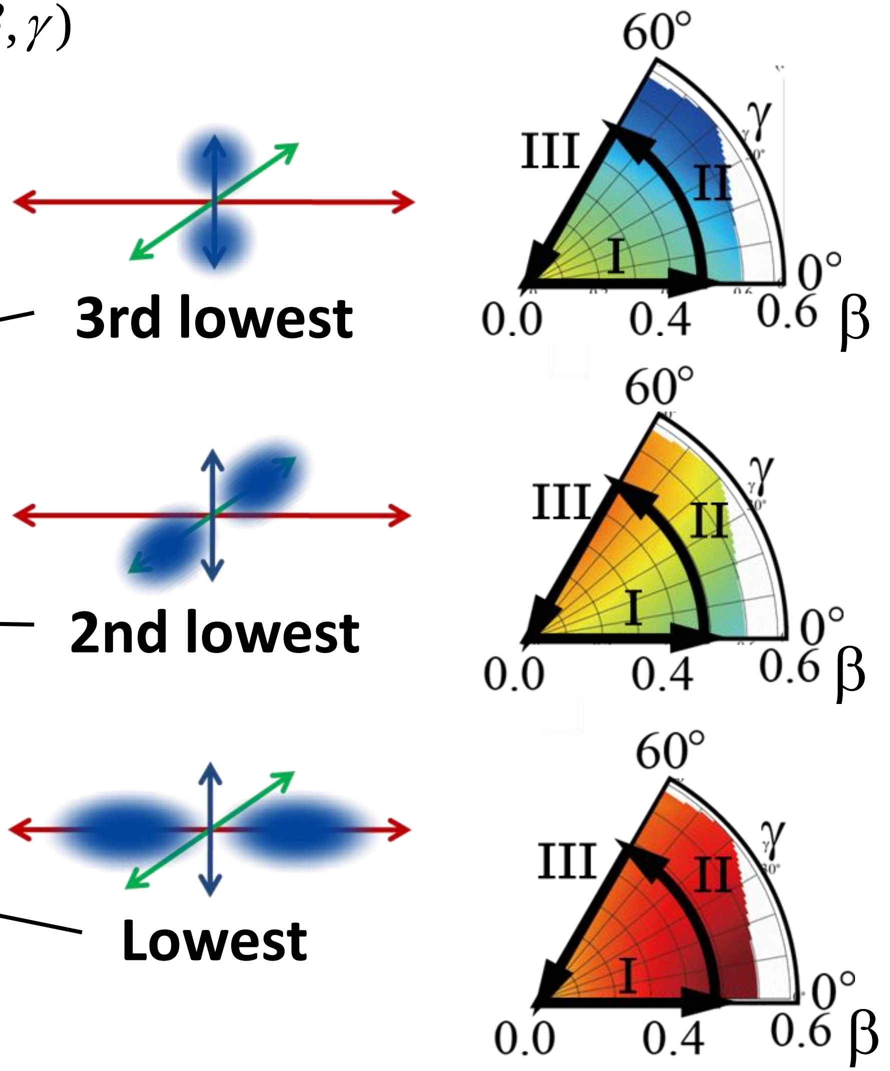
Single particle energy of Λ particle is different in each p state corresponding the difference of overlap between Λ and nucleons

Results： Single particle energy of Λ hyperon ε_Λ

$^{27}_{\Lambda}$Mg (AMD)

$$\varepsilon_\Lambda(\beta,\gamma)=E_{\Lambda p}(\beta,\gamma)-E_{core}(\beta,\gamma)$$

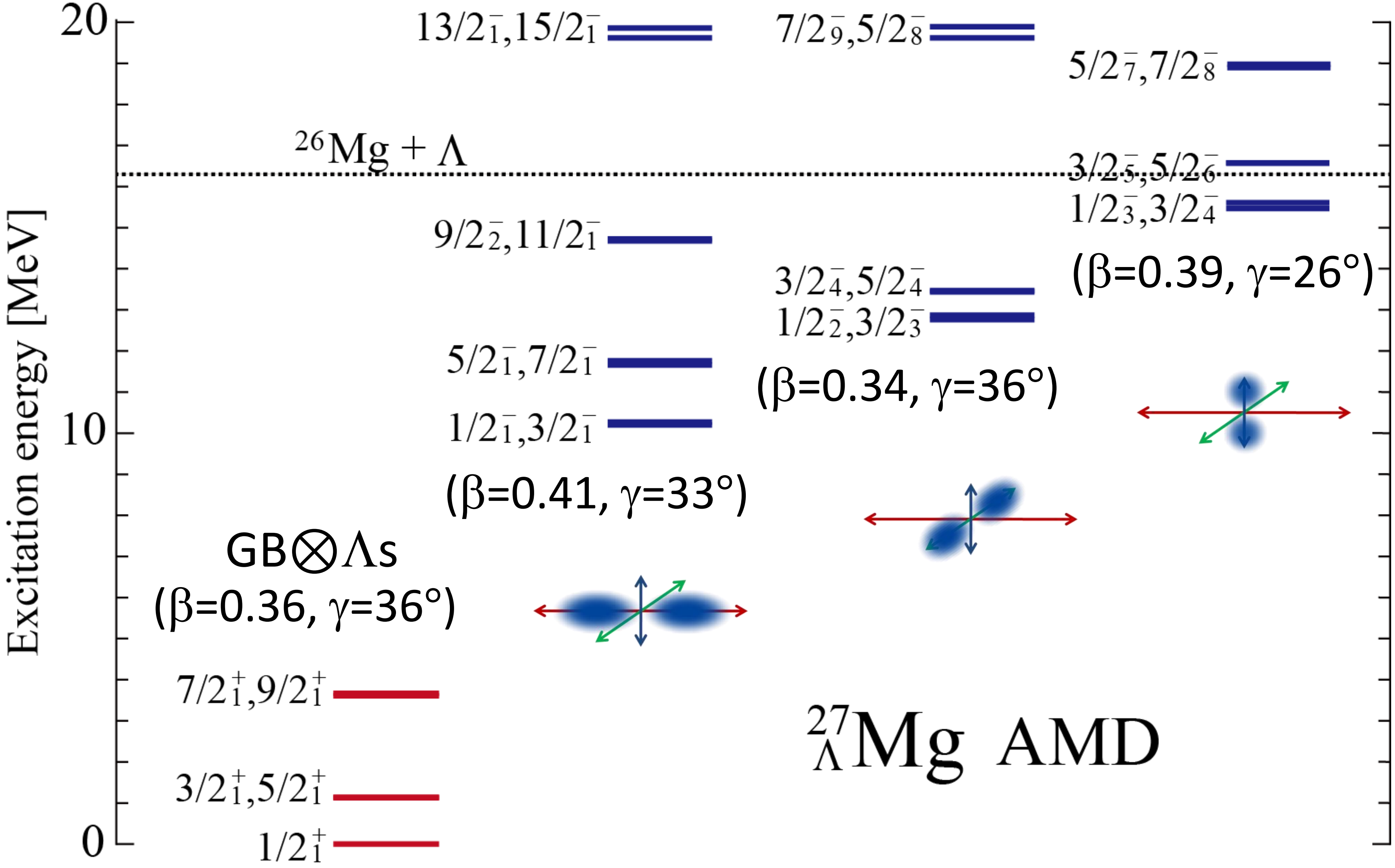




3 different p-states appear with triaxial deformation

Results: Excitation spectra

- **3 bands are obtained by Λ in *p*-orbit** → **Splitting of the *p* states**



cf. ^{26}Mg ground state:
β=0.41, γ=33°

Λ particle as a probe to study triaxial deformation

◆Advantages of using hypernuclei (including future tasks)

- **3 different p states could be a direct evidence**

  → Production cross section

- **Deformation changes if occur**

  → Λ does not affect triaxiality in p state

  Λ in s orbit can reduce β deformation? State dependence?

- **Difference of (β,γ) deformation in ground- and excited states**
- **Is it possible to see β, γ-soft nature of energy surface?**

Summary and Future problems

◆Hypernuclear deformation

- **Deformation change by Λ particle**
- **Difference of B_Λ depending on deformation**
- **Coupling of Λ to deformed nuclei**
  - Today's topic: *p*-states in triaxially deformed Mg hypernuclei

◆Study of ^{26}Mg: possibility to use Λ as a probe of deformation

- **Detailed analysis: βγ-dep., rotational bands, B(E2), … etc.**
- **Production cross section: how to identify?**
- **Can Λ be a probe to study (β)γ-soft nature?**